

1953

Feedback and stability of junction transistor circuits

Donald William Gade
Iowa State College

Follow this and additional works at: <https://lib.dr.iastate.edu/rtd>

 Part of the [Electrical and Electronics Commons](#)

Recommended Citation

Gade, Donald William, "Feedback and stability of junction transistor circuits " (1953). *Retrospective Theses and Dissertations*. 12418.
<https://lib.dr.iastate.edu/rtd/12418>

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.

NOTE TO USERS

This reproduction is the best copy available.

UMI[®]

FEEDBACK AND STABILITY OF JUNCTION
TRANSISTOR CIRCUITS

by

Donald William Gade

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Electrical Engineering

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State College

1953

UMI Number: DP11817

INFORMATION TO USERS

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleed-through, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

UMI[®]

UMI Microform DP11817

Copyright 2005 by ProQuest Information and Learning Company.

All rights reserved. This microform edition is protected against unauthorized copying under Title 17, United States Code.

ProQuest Information and Learning Company
300 North Zeeb Road
P.O. Box 1346
Ann Arbor, MI 48106-1346

TABLE OF CONTENTS

	Page
I. INTRODUCTION	1
II. REVIEW OF LITERATURE	4
III. TRANSISTOR EQUIVALENT CIRCUITS	7
A. Introduction to Equivalent Circuits	7
B. Mesh-derived Equivalent Circuits	11
C. Nodal-derived Equivalent Circuits	15
D. Evaluation of the Equivalent Circuit Parameters	20
IV. SINGLE STAGE TRANSISTOR CIRCUITS	25
A. Development of Feedback Circuits	25
B. Selection of Circuit Terminations	33
C. Analysis of Circuits Containing Series or Parallel Feedback	35
1. The general transistor circuit	35
2. The grounded base transistor circuit	41
3. The grounded emitter transistor circuit	48
4. The grounded collector transistor circuit	54
D. Analysis of Circuits Containing Both Series and Parallel Feedback	56

TABLE OF CONTENTS (continued)

	Page
V. CASCADED TRANSISTOR CIRCUITS	60
A. Equivalent Circuits for Cascaded Transistors	60
B. Analysis of Cascaded Circuits Containing Series or Parallel Feedback	66
C. Analysis of Cascaded Transistor Circuits Containing Both Series and Parallel Feedback	71
VI. SUMMARY	74
VII. SELECTED REFERENCES	76
VIII. ACKNOWLEDGMENTS	89
IX. APPENDICES	90
APPENDIX A. MEASUREMENT OF TRANSISTOR NETWORK COEFFICIENTS	91
APPENDIX B. RETURN DIFFERENCE AND SENSITIVITY FOR TRANSISTORS	96

I. INTRODUCTION

During the period from 1940 to the present time, a considerable advancement was made in the field of semiconductors. In particular, the properties of silicon and germanium were studied. From this, detailed information was obtained about the energy levels and the conducting properties of semiconductors.

As evidence of the advancement, it was announced in 1948 that a radically new type of amplifying device had been discovered¹. This amplifying device was called a point-contact transistor. Its ability to amplify was based upon the conducting properties of the semiconductor germanium.

Tremendous progress has been made in the field of transistors since their announcement. To the rapidly growing list have been added the coaxial transistor, the junction transistor, the field-effect transistor, the transistor tetrode, and the P-N-P-N

¹J. Bardeen and W. H. Brattain. Transistor, a semiconductor triode. Phys. Rev. 74: 230-231. 1948.

W. H. Brattain and J. Bardeen. Nature of the forward current in germanium point contacts. Phys. Rev. 74: 231-232. 1948.

W. Shockley and G. L. Pearson. Modulation of conductance of thin films of semiconductors by surface charges. Phys. Rev. 74: 232-233. 1948.

transistor. Manufacturing techniques have been developed and improved to the point where point-contact and junction transistors are now available in quantity. Transistor circuitry has been developed to the point where only the present cost (and possibly quality) of transistors prevents them from replacing many vacuum tubes. New fields have been found for transistors where amplifying devices had not previously been used.

At present, it appears that the junction transistor holds the greatest possibilities outside the field of switching circuits. It has many natural advantages over other transistor types: the theoretical aspects of the junction transistor are easily understood; its electrical characteristics can be accurately predicted; it is capable of producing relatively large power gains; as an amplifier, it is short-circuit stable; it is rugged; its power requirements are extremely small; its size makes it particularly adaptable to miniaturized circuits.

In addition to the above advantages, the availability of N-P-N and P-N-P transistors makes it possible to take advantage of complementary symmetry principles. Using these principles, biasing problems are simplified and circuit-component requirements are reduced. In push-pull stages, complementary symmetry can be used to eliminate the need of a phase inverter.

Single unit symmetry was introduced by junction transistors. Using this principle, it is possible to reverse the direction of

amplification by reversing bias voltages.

As indicated by these present advantages, the junction transistor is of great importance. This thesis pertains to junction transistors; specifically, it is concerned with the application of feedback to transistor circuits with the resulting problem of circuit analysis.

II. REVIEW OF LITERATURE

Literally hundreds of articles concerning transistors have appeared in the physics journals and the electrical engineering periodicals since 1948. These articles pertain to all phases of the transistor including theory, circuit properties, and applications. Two books have been written dealing exclusively with transistors¹. However, the amount of information available concerning transistor feedback circuits and their analysis is rather limited.

R. L. Wallace, Jr. and W. J. Pietenpol² published one of the first articles on junction transistors in 1951, discussing circuit properties and applications of single stage N-P-N transistors. This discussion was limited to the use of one equivalent circuit for all connections, and was primarily concerned with basic circuit properties without reference to feedback aspects.

¹W. Shockley. Electrons and holes in semiconductors. 1st ed. D. Van Nostrand Company, Inc. 1950.

R. F. Shea. Principles of transistor circuits. 1st ed. John Wiley and Sons, Inc. 1953.

²R. L. Wallace, Jr. and W. J. Pietenpol. Some circuit properties and applications of N-P-N transistors. Bell Sys. Tech. Jour. 30: 530-563. 1951.

Richard F. Shea¹ considered feedback as a means of stabilizing junction transistor operating points. He investigated the effect of adding series resistors to the transistor terminals in order to reduce the variation in the collector saturation current. This was a very important contribution. Stabilization methods introduced by him are used in most present day circuit designs.

D. E. Thomas² studied the effect of feedback on cutoff frequency³. He considered only the feedback inherently present in the transistor. He concluded that return difference⁴ could be used as a measure of circuit cutoff frequency.

A comprehensive study was made on transistor equivalent circuits and associated terminology by L. J. Giacoletto⁵. He attempted to classify transistor equivalent circuits and to standardize the

¹Richard F. Shea. Transistor operation: stabilization of operating points. Proc. of the IRE. 40: 1435-1437. 1952.

²D. E. Thomas. Transistor amplifier-cutoff frequency. Proc. of the IRE. 40: 1481-1483. 1952.

³Cutoff frequency is defined as the frequency at which the gain is 3db below the low frequency gain.

⁴See Appendix B for definition of return difference for transistors.

⁵L. J. Giacoletto. Terminology and equations for linear active four-terminal networks including transistors. RCA Review. 14: 28-46. 1953.

subscript notation for the network coefficients and the equivalent circuit parameters. It is possible that his notation will be standardized by the professional groups.

In a recent article by Peter G. Sulzer¹, junction transistor circuit applications were reviewed. He illustrated several single-stage and cascaded transistor circuits which he termed "building-block" circuits. All of these circuits were of the grounded emitter connection. For d-c stabilization, he used a resistor in series with the emitter, a resistor between base and collector, and a resistor between base and ground. This thesis identifies feedback of this type as series and parallel feedback.

¹Peter G. Sulzer. Junction transistor circuit applications. Electronics. 26: 170-173. Aug. 1953.

III. TRANSISTOR EQUIVALENT CIRCUITS

A. Introduction to Equivalent Circuits

As the first step in analyzing the linear operation of transistors at low frequencies, suitable equivalent circuits must be selected. Many different transistor equivalent circuits are available¹. In order to prevent confusion, it is necessary to derive the equivalent circuits needed.

¹J. M. Early. Effects of space-charge layer widening in junction transistors. *Proc. of the IRE.* 40: 1401-1406. 1952.

L. J. Giacoletto. Junction transistor equivalent circuits and vacuum tube analogy. *Proc. of the IRE.* 40: 1490-1493. 1952.

L. J. Giacoletto. Terminology and equations for linear active four-terminal networks including transistors. *RCA Review.* 14: 28-46. 1953.

Geoffrey Knight, Jr., Richard A. Johnson, and Roland B. Holt. Measurement of the small signal parameters of transistors. *Proc. of the IRE.* 41: 983-989. 1953.

R. M. Ryder and R. J. Kirchner. Some circuit aspects of the transistor. *Bell Sys. Tech. Jour.* 28: 367-401. 1949.

Wallace and Pietsenpol, *op. cit.*

R. L. Wallace and G. Raisbeck. Duality as a guide in transistor circuit design. *Bell Sys. Tech. Jour.* 30: 381-417. 1951.

Equivalent circuits may be derived from the mesh or nodal equations of a general three-terminal network¹. This immediately divides the equivalent circuits into two distinct groups: those equivalent circuits derived on the basis of mesh equations taking the form of a T and those equivalent circuits derived on the basis of nodal equations taking the form of a π . It will be convenient to first obtain a general equivalent circuit of each group, and from these build up specific circuits.

The general solution to the first group can be found from the mesh equations of the active three-terminal network of Figure 1.

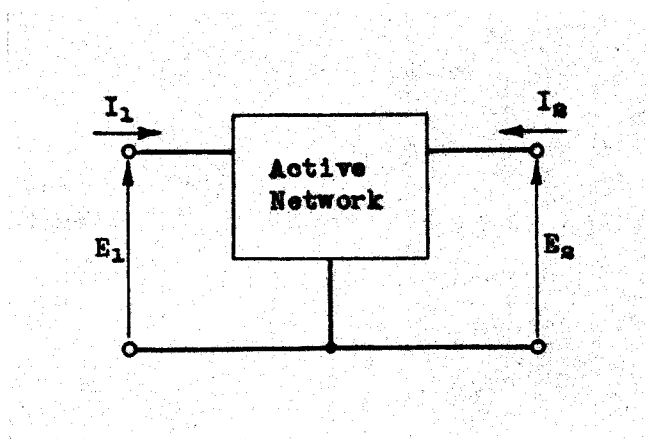


Figure 1. Active three-terminal network²

¹L. C. Peterson. Equivalent circuits of linear active four-terminal networks. Bell Sys. Tech. Jour. 27: 593-622. 1948.

²The directions chosen for voltages and currents in this figure are standard nomenclature and will be used throughout this manuscript.

At low frequencies, where the network coefficients are real,

$$E_1 = r_{11}I_1 + r_{12}I_2, \quad (1)$$

and

$$E_2 = r_{21}I_1 + r_{22}I_2. \quad (2)$$

These equations may be written in the form

$$E_1 = r_{11}I_1 + r_{12}I_2, \quad (3)$$

and

$$E_2 = r_{12}I_1 + r_{22}I_2 + (r_{21}-r_{12})I_1. \quad (4)$$

Equations (3) and (4) suggest the equivalent circuit of Figure 2.

In this figure, the network coefficients are related to the equivalent circuit parameters as follows:

$$r_{11} = r_1 + r_2,$$

$$r_{12} = r_2,$$

$$r_{21} = r_4 + r_2,$$

(5)

and

$$r_{22} = r_2 + r_3.$$

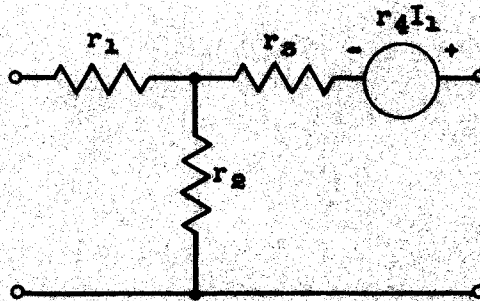


Figure 2. General mesh-derived transistor equivalent circuit

Because the connections for the transistor of Figure 1 are not specified, the equivalent circuit of Figure 2 can be used for any connection. Of the six possible connections, only three produce appreciable power gain; grounded base with emitter input and collector output, grounded emitter with base input and collector output, and grounded collector with base input and emitter output. These connections will be referred to as the grounded base, the grounded emitter, and the grounded collector, respectively. No other connection will be discussed.

The same procedure can be repeated with the nodal equations for the three-terminal network of Figure 1. The equations are

$$I_1 = g_{11}E_1 + g_{12}E_2, \quad (6)$$

and

$$I_2 = g_{21}E_1 + g_{22}E_2. \quad (7)$$

Here g 's are used to indicate that the admittances are real¹.

Rearrangement of equations (6) and (7) gives

$$I_1 = g_{11}E_1 + g_{12}E_2, \quad (8)$$

and

$$I_2 = g_{12}E_1 + g_{22}E_2 + (g_{21} - g_{12})E_1. \quad (9)$$

¹The g 's used here are simplifications of admittance and should not be confused with the network coefficient resulting when I_1 and E_2 are considered independent.

Equations (8) and (9) suggest the equivalent circuit of Figure 3.

The network coefficients and equivalent circuit parameters for this

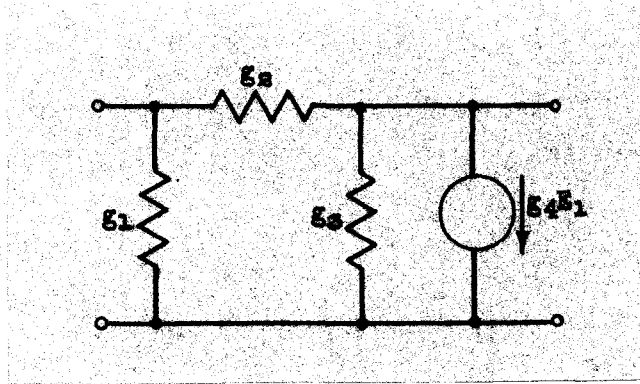


Figure 3. General nodal-derived transistor equivalent circuit

circuit are related as follows:

$$\epsilon_{11} = \epsilon_1 + \epsilon_2 ,$$

$$\epsilon_{12} = -\epsilon_2 ,$$

$$\epsilon_{21} = \epsilon_4 - \epsilon_2 ,$$

(10)

and

$$\epsilon_{22} = \epsilon_2 + \epsilon_3 .$$

The equivalent circuit of Figure 3 can be used for any connection.

B. Mesh-Derived Equivalent Circuits

One of the first connections used with transistors was the

grounded base connection¹, with the equivalent circuit of Figure 4. This is probably the most familiar equivalent circuit at present. The parameter subscripts were chosen to correspond to the actual terminal designations as shown. The subscript m on r is proper because it comes from a mutual impedance term similar to g_m in vacuum tubes.

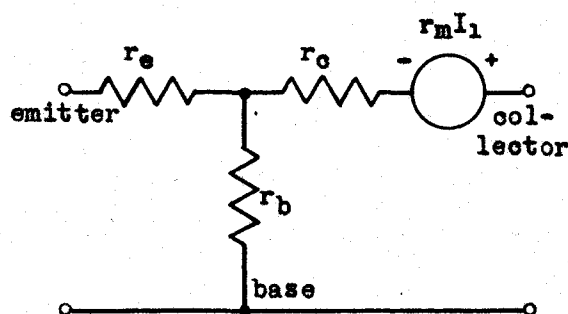


Figure 4. Mesh-derived equivalent circuit for grounded base

Since the equivalent circuit of Figure 4 may be used in any of the three transistor connections by exchange of terminals and proper identification of the voltage of the equivalent generator, it may appear that only this particular equivalent circuit is necessary.

¹Ryder and Kirchner, op. cit.

However, convenience in circuit analysis suggests that the voltage of the active element be always in terms of the input current I_1 . Consequently, two more mesh-derived equivalent circuits can be conveniently used.

The equivalent circuit fitting the general form of Figure 2, for the grounded emitter connection, can be found without introducing new parameters. The equations for the circuit of Figure 4 connected as grounded emitter are

$$E_1 = (r_b + r_e)I_1 + r_e I_2 \quad (11)$$

and

$$E_2 = r_e I_1 + (r_c + r_e)I_2 - r_m(I_1 + I_2) \quad (12)$$

Upon rearranging these equations, we have

$$E_1 = (r_b + r_e)I_1 + r_e I_2 \quad (13)$$

and

$$E_2 = r_e I_1 + (r_c - r_m + r_e)I_2 - r_m I_1 \quad (14)$$

Equations (13) and (14) suggest an equivalent circuit which fits the general form and which is shown in Figure 5. Besides having the active element in terms of the input base current, this equivalent circuit has the further advantage of not introducing new parameters.

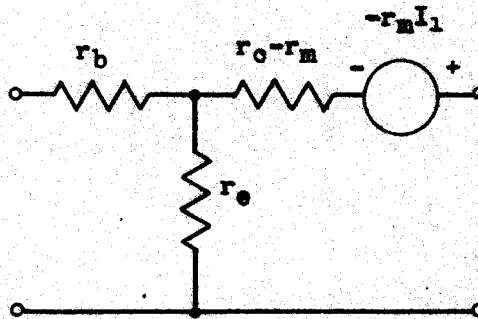


Figure 5. Mesh-derived equivalent circuit for grounded emitter

The equations for the circuit of Figure 4 connected as grounded collector are

$$E_1 = (r_b + r_c)I_1 + (r_c - r_m)I_2, \quad (15)$$

and

$$E_2 = r_c I_1 + (r_c + r_e)I_2 - r_m I_2. \quad (16)$$

Upon rearranging these equations as

$$E_1 = (r_b + r_c)I_1 + (r_c - r_m)I_2, \quad (17)$$

and

$$E_2 = (r_c - r_m)I_1 + (r_c - r_m + r_e)I_2 + r_m I_1, \quad (18)$$

the circuit of Figure 6 is suggested, with the same advantages as those of Figure 5.

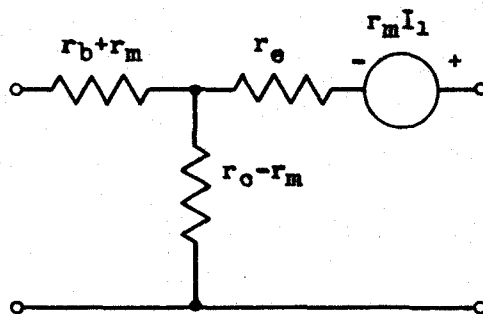


Figure 6. Mesh-derived equivalent circuit for grounded collector

C. Nodal-Derived Equivalent Circuits

The second group of transistor equivalent circuits comprise those based upon the nodal equations. Circuits of this type were first introduced by L. J. Giacoletto¹ of the RCA Laboratories. The circuits proposed by him involve a complex system of notation based upon three subscripts for each coefficient. Although the system is foolproof, its practicability is questionable.

One way to simplify the notation is to write the parameters

¹L. J. Giacoletto. Terminology and equations for linear active four-terminal networks including transistors. RCA Review. 14: 28-46. 1953.

of the nodal-derived equivalent circuit in terms of the parameters used for the mesh-derived equivalent circuits. This method will be used throughout this manuscript.

The nodal-derived equivalent circuit for the grounded base connection may be easily obtained from the circuit equations of Figure 4. These equations are

$$E_1 = (r_e + r_b)I_1 + r_b I_2, \quad (19)$$

and

$$E_2 = (r_b + r_m)I_1 + (r_c + r_b)I_2. \quad (20)$$

Solving these two equations for I_1 and I_2 in terms of E_1 and E_2 and rearranging the results gives

$$I_1 = \frac{r_c + r_b}{|r|} E_1 - \frac{r_b}{|r|} E_2, \quad (21)$$

and

$$I_2 = -\frac{r_b}{|r|} E_1 + \frac{r_c + r_b}{|r|} E_2 - \frac{r_m}{|r|} E_1, \quad (22)$$

where

$$|r| = (r_e + r_b)(r_c + r_b) - r_b(r_b + r_m) > 0. \quad (23)$$

Equations (21) and (22) suggest the equivalent circuit shown in Figure 7. This circuit fits the general nodal-derived equivalent circuit of Figure 3. It has the advantage of having the current of the active element in terms of the input voltage E_1 .

The grounded emitter equivalent circuit can be obtained in a similar manner. Solving for I_1 and I_2 from Equations (11) and (12)

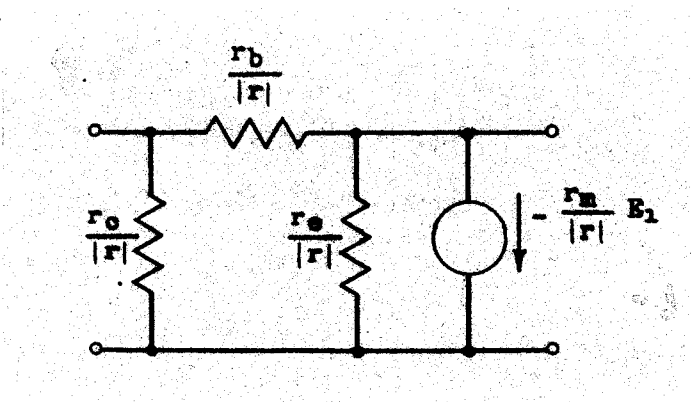


Figure 7. Nodal-derived equivalent circuit for the grounded base

results in

$$I_1 = \frac{(r_c - r_m + r_e)}{|r|} E_1 - \frac{r_e}{|r|} E_2, \quad (24)$$

and

$$I_2 = -\frac{r_e}{|r|} E_1 + \frac{(r_b + r_e)}{|r|} E_2 + \frac{r_m}{|r|} E_1. \quad (25)$$

Equations (24) and (25) suggest the circuit of Figure 8.

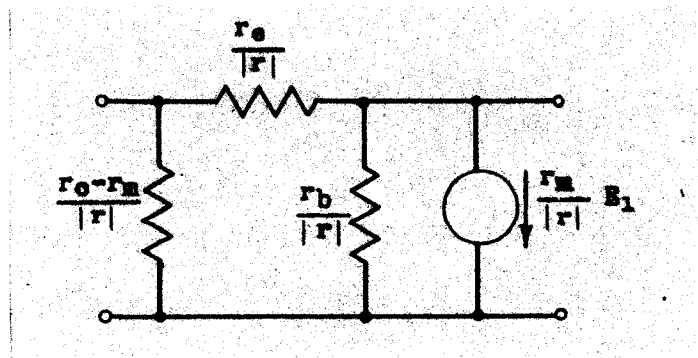


Figure 8. Nodal-derived equivalent circuit for the grounded emitter

Solution for I_1 and I_2 from Equations (15) and (16) produces the equations

$$I_1 = \frac{r_c - r_m + r_e}{|r|} E_1 - \frac{r_c - r_m}{|r|} E_2, \quad (26)$$

and

$$I_2 = -\frac{r_c - r_m}{|r|} E_1 + \frac{r_b + r_c}{|r|} - \frac{r_m}{|r|} E_2. \quad (27)$$

Equations (26) and (27) suggest the equivalent circuit of Figure 9.

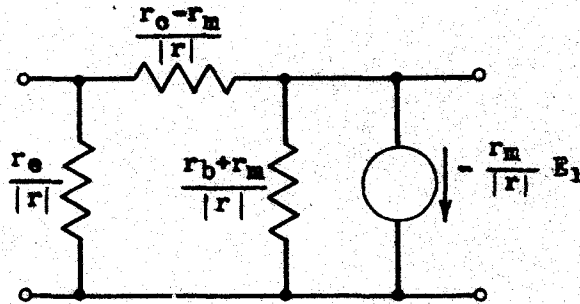
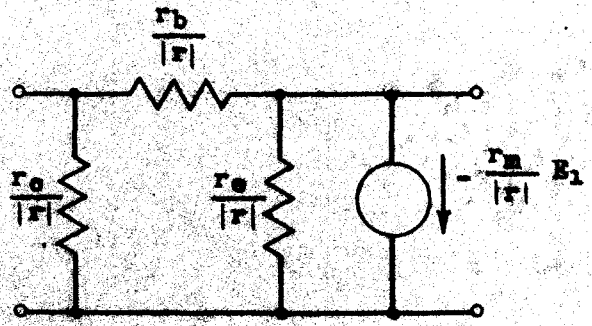
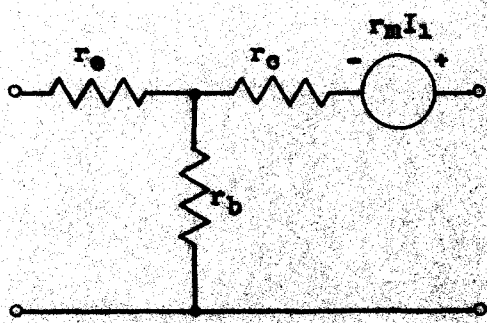


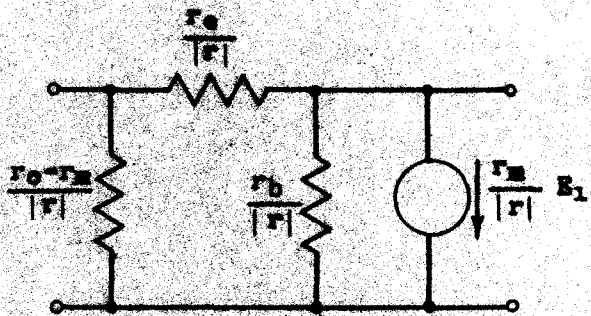
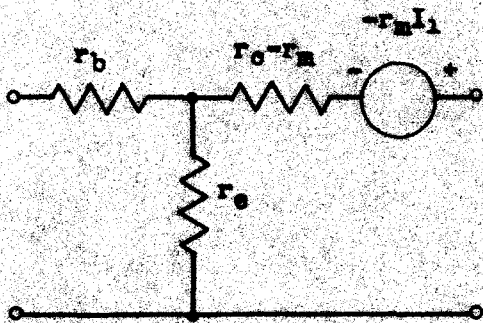
Figure 9. Nodal-derived equivalent circuit for the grounded collector stage

It should be noted that an equivalent circuit is not limited to the particular kind of transistor connection for which it was derived. Each equivalent circuit could be used in any of the connections. This is true for both the mesh and nodal-derived circuits.

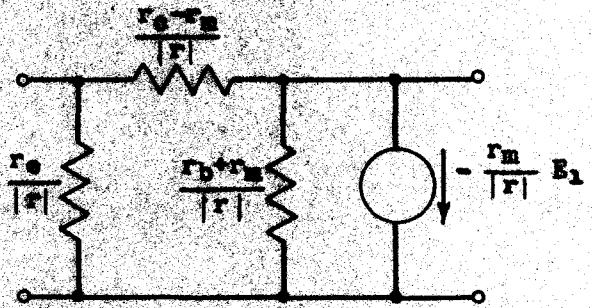
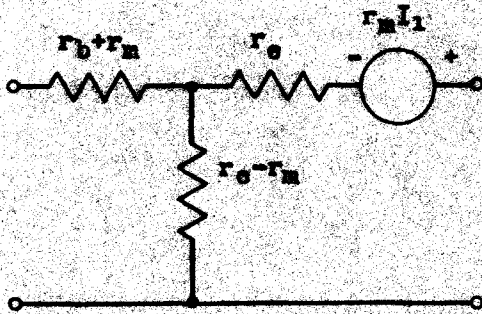
Figure 10 summarizes the work of this section and is included merely for convenience.



Grounded base



Grounded emitter



Grounded collector

Mesh-derived

Nodal-derived

Figure 10. Transistor equivalent circuits

D. Evaluation of the Equivalent Circuit Parameters

Having selected the transistor equivalent circuits, it now becomes necessary to evaluate the equivalent circuit parameters. This is easily done through the network coefficients pertaining to the mesh or nodal equations.

In general, knowing the network coefficients for one type of connection of the transistor, it is possible to calculate the network coefficients for any other connection¹. This is particularly adaptable to the equivalent circuits of Figure 10. In these circuits, all network coefficients are written in terms of the equivalent circuit parameters of the mesh-derived grounded base stage. This is summarized in Table II. If the network coefficients for any one connection are known, the equivalent circuit parameters r_e , r_b , r_c and r_m can be evaluated. Knowing these parameters, any equivalent circuit can be easily obtained.

To evaluate the transistor parameters, eight selected P-N-P junction transistors were connected as grounded emitter and the network coefficients pertaining to the nodal equations were measured². All measurements were made on the General Radio Type

¹L. J. Giacoletto. Terminology and equations for linear active four-terminal networks including transistors. RCA Review. 14: 28-46. 1953.

Guillemin, op. cit.

²See Appendix A.

Table I
Experimental Transistor Network Coefficients

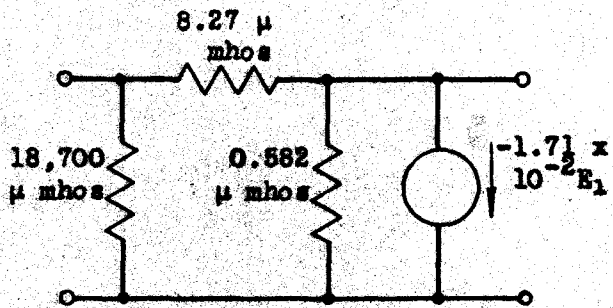
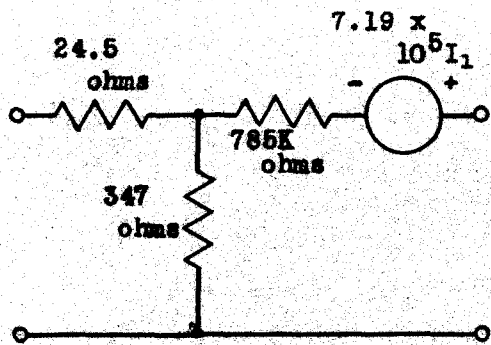
P-N-P Transistor Type	D-C Bias Values					Network Coefficients*					
	V ₁ volts	V ₂ volts	I ₁ μa	I ₂ ma	I ₃ μa	g ₁₁ μ mho	-g ₁₂ μ mho	-g ₁₃ /g ₁₁ x 10 ⁶	g ₂₂ μ mho	-g ₂₁ μ mho	-g ₂₁ /g ₂₂ ---
1 CK721	0.16	-5.0	11	-1.0	782	2.65	3400	18.1	-31,300	-1740	
2 CK721	0.16	-5.0	61	-1.0	1710	0.264	154	8.55	-30,600	-3910	
3 CK721	0.165	-5.0	29	-1.0	704	0.291	405	8.48	-24,200	-2990	
4 CK721	0.18	-5.0	70	-1.0	2170	1.074	505	8.00	-32,100	-4350	
5 ^φ CK722	0.19	-5.0	82	-1.0	1570	0.582	384	8.85	-17,100	-2030	
6 CK722	0.17	-5.0	51	-1.0	2400	5.40	2220	31.0	-25,200	-821	
7 CK722	0.14	-5.0	1	-1.0	1100	15.7	14,300	72.5	-32,500	-464	
8 CK722	0.18	-5.0	63	-1.0	2060	1.82	876	13.0	-26,700	-2060	

*Transistor connected as grounded emitter

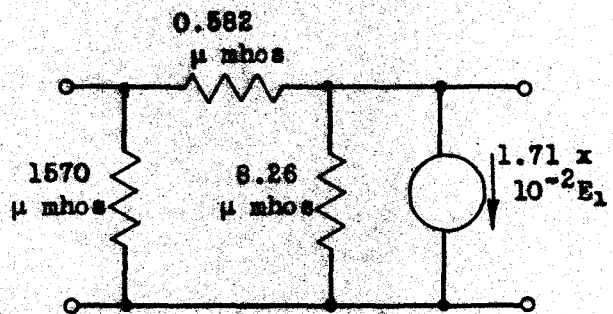
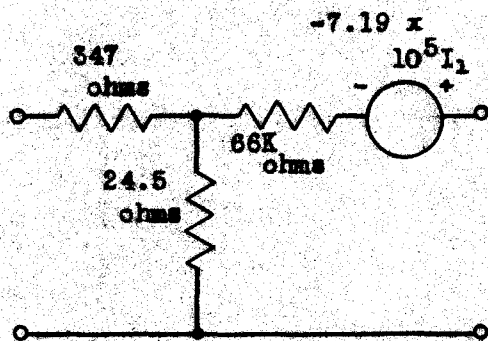
^φSelected as a typical transistor

Table II
Summary of Network Coefficients

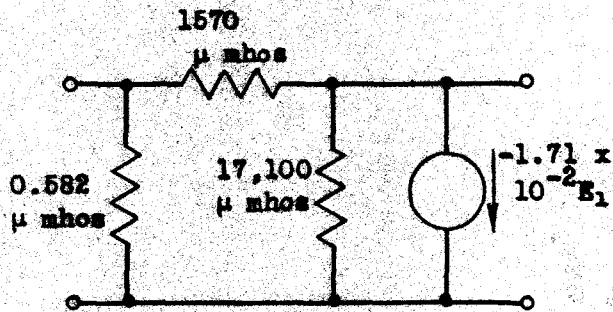
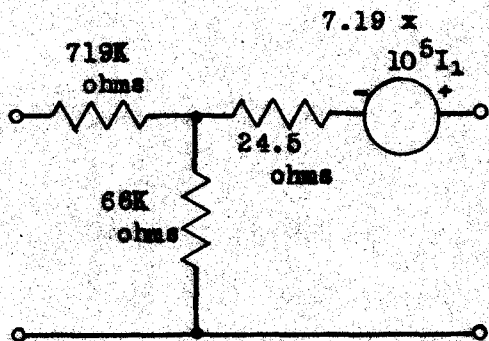
Coefficient	Connection		
	Grounded Base	Grounded Emitter	Grounded Collector
	Formula	Formula	Formula
	Value	Value	Value
Mesh derived	(ohm)	(ohm)	(ohm)
r_{11}	$r_e + r_b$	$r_b + r_e$	$r_b + r_e$
	372	372	785,000
r_{12}	r_b	r_e	$r_e - r_m$
	347	24.5	66,000
r_{21}	$r_b + r_m$	$r_e - r_m$	r_e
	719,000	-719,000	785,000
r_{22}	$r_e + r_b$	$r_e - r_m + r_e$	$r_e - r_m + r_e$
	785,000	66,000	66,000
Modal derived	(μ mho)	(μ mho)	(μ mho)
g_{11}	$\frac{r_e + r_b}{ r }$	$\frac{r_e - r_m + r_e}{ r }$	$\frac{r_e - r_m + r_e}{ r }$
	18,700	1,570	1,570
g_{12}	$-\frac{r_b}{ r }$	$-\frac{r_e}{ r }$	$-\frac{r_e - r_m}{ r }$
	-8.27	-0.582	-1,570
g_{21}	$-\frac{r_b + r_m}{ r }$	$-\frac{r_e - r_m}{ r }$	$-\frac{r_e}{ r }$
	-17,100	17,100	-18,700
g_{22}	$\frac{r_e + r_b}{ r }$	$\frac{r_b + r_e}{ r }$	$\frac{r_b + r_e}{ r }$
	8.85	8.85	-18,700



Grounded base



Grounded emitter



Grounded collector

Mesh-derived

Nodal-derived

Figure 11. Equivalent circuits for a typical junction transistor

561-D, Vacuum Tube Bridge. In each case the collector to emitter voltage was set at five volts d-c and the d-c base to emitter voltage was varied to give a collector current of one milliampere. This was considered to be a typical bias point.

Table I summarizes the results obtained in measuring the network coefficients. It will be noted that considerable variation exists in the value of corresponding coefficients. Type CK721 transistors give slightly higher power gains than do type CK722 transistors, however, the dividing line is somewhat obscure.

With the help of published material and manufacturer's information, it was decided that the type CK722 transistor of line 5, Table I would be considered as the typical transistor¹. Therefore, all calculations requiring values are made using the measured values of this transistor.

Using the measured network coefficients of the typical transistor of Table I, the equivalent circuit parameters r_e , r_b , r_c and r_m were calculated. From these calculated values, the network coefficients corresponding to other equivalent circuits were evaluated. Table II summarizes these.

From the evaluated network coefficients of Table II, the corresponding equivalent circuits for the typical transistor were drawn and summarized in Figure 11.

¹Raytheon Manufacturing Company. Germanium transistor data sheets No. CS-2881 and No. CS-2884. 55 Chapel Street, Newton 58, Massachusetts. Nov. 1952.

IV. SINGLE STAGE TRANSISTOR CIRCUITS

A. Development of Feedback Circuits

The transistor is inherently a feedback device. This may be seen by considering the general mesh-derived equivalent circuit of Figure 12. Here it is assumed that the transistor is supplied by a

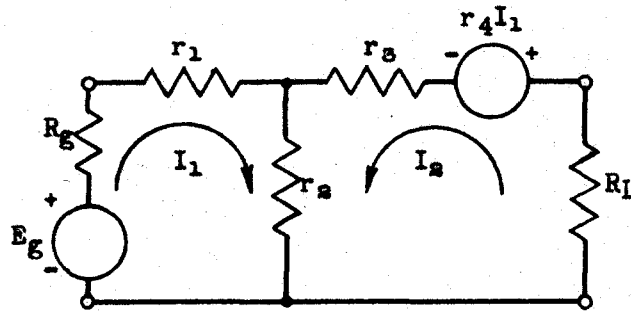


Figure 12. General mesh-derived equivalent circuit with terminations

generator E_g of internal resistance R_g and terminated in the resistance R_L . The expression for return difference¹ for r_4 is

¹See Appendix B.

H. W. Bode. Network analysis and feedback amplifier design. 1st ed. D. Van Nostrand Co., Inc. 1945.

Thomas, op. cit.

$$F = 1 - \frac{r_2 r_4}{(R_g + r_1 + r_2)(r_2 + r_3 + R_L) - r_2^2} \quad (28)$$

In terms of the network coefficients, equation (28) may be written as

$$F = 1 - \frac{r_{12}(r_{21} - r_{12})}{(R_g + r_{11})(r_{22} + R_L) - r_{12}^2} \quad (29)$$

It is evident from equation (29) that feedback may be negative or positive depending upon the sign of r_{21} . Furthermore, feedback could be eliminated if r_{12} could be eliminated. Manufacturing techniques can be used to control r_{12} but not to eliminate it¹.

Similar reasoning can be applied to the general nodal-derived equivalent circuit. In this case, it can be shown that feedback is due to the presence of the g_{12} coefficient.

Feedback in a transistor circuit may be controlled through the r_{12} or g_{12} coefficient. Any circuit additions which change these coefficients also change the feedback. Additions of this kind will be referred to as added feedback, to distinguish between that feedback which is inherently present and that which is added.

¹Abraham Coblentz and Harry L. Owens. Variation of transistor parameters with temperature. Proc. of the IRE. 40: 1472-1476. 1952.

Early, op. cit.

J. A. Merton. Present status of transistor development. Proc. of the IRE. 40: 1314-1326. 1952.

The circuit of Figure 13(a) represents a straightforward method of increasing feedback. A mesh-derived equivalent circuit is shown because the type of feedback used is most easily handled by mesh equations. Writing the equations for this circuit without terminations gives

$$E_1 = (r_1 + r_2 + R_2)I_1 + (r_2 + R_2)I_2 , \quad (30)$$

and

$$E_2 = (r_4 + r_2 + R_2)I_1 + (r_2 + r_3 + R_2)I_2 . \quad (31)$$

Here, standard four-terminal network notation is used as shown in Figure 1. Considering the new circuit element R_2 to be a part of the equivalent circuit, we can define the new network coefficients to be

$$\begin{aligned} R_{11} &= r_1 + r_2 + R_2 , \\ R_{12} &= r_2 + R_2 , \\ R_{21} &= r_4 + r_2 + R_2 , \end{aligned} \quad (32)$$

and

$$R_{22} = r_2 + r_3 + R_2 .$$

From the relationships of Equation (32) we see that R_2 is added to each of the network coefficients and that R_{12} (now replacing r_{12}) cannot be controlled independently.

Feedback can also be controlled by the use of transformer coupling between loops as shown by Figures 13(b) and 13(c). In this case the transformer can be connected either for positive-added

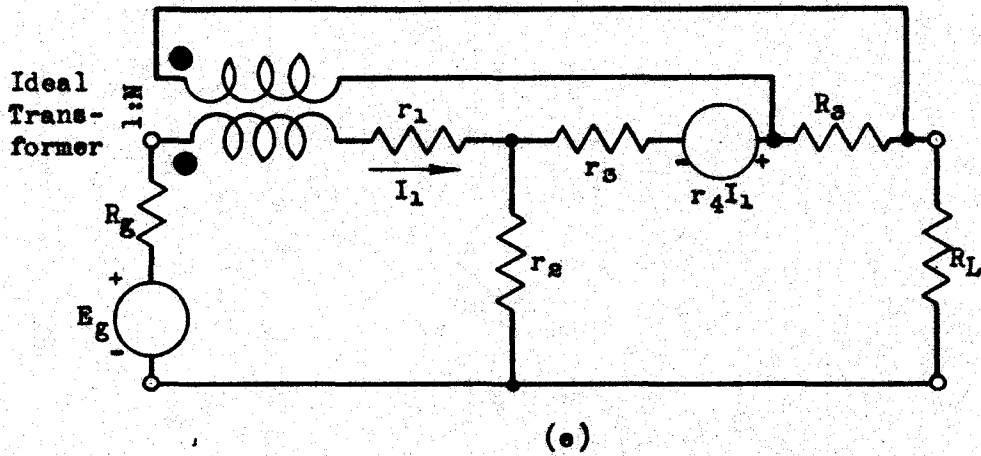
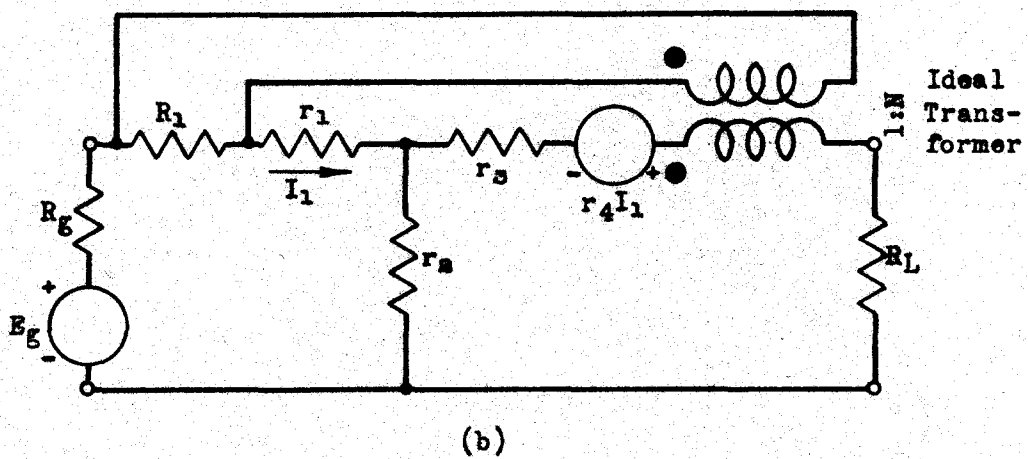
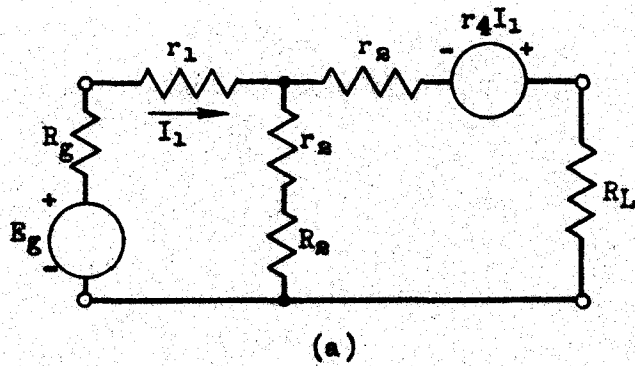


Figure 13. Series transistor feedback circuits

or for negative-added feedback. Because the transistor circuit contains feedback originally, transformer coupling can be used to enhance, lessen, or change this feedback from positive to negative or negative to positive as the case may be. Referring to the network coefficients for Figures 13(b) and 13(c) listed in Table III, R_{12} is increased, decreased, or changed in sign depending upon the size of the feedback resistor (R_1 or R_3) introduced and the turns ratio of the transformer. It should be noted that R_{11} or R_{22} does not change when the transformer connections are reversed.

The feedback circuits of Figure 13 require a resistance to be placed in series with r_1 , r_2 , or r_3 . Therefore, we shall call these circuits "series feedback circuits". This should not be confused with series feedback circuits as defined by Bode¹.

Feedback circuits which are most easily handled by use of the nodal-derived equivalent circuits are shown in Figure 14. These circuits will be called "parallel feedback circuits" because added feedback requires a conductance to be placed in parallel with g_1 , g_2 , or g_3 .

Table IV summarizes the network coefficients for the parallel feedback circuits. It will be noted that these network coefficients

¹Bode, op. cit.

Table III
Network Coefficients for Series Feedback Circuits

Circuit	Transformer Connection	Feedback Element	Network Coefficient			
			R_{11}	R_{12}	R_{21}	R_{22}
Figure 13(a)	----	R_2	$r_1 + r_2 + R_2$	$r_2 + R_2$	$r_4 + r_2 + R_2$	$r_2 + r_3 + R_2$
Figure 13(b)	As shown	R_1	$r_1 + r_2 + R_1$	$r_2 + \frac{R_1}{N}$	$r_4 + r_2 + \frac{R_1}{N}$	$r_2 + r_3 + \frac{R_1}{N^2}$
	Reversed	R_1	$r_1 + r_2 + R_1$	$r_2 - \frac{R_1}{N}$	$r_4 + r_2 - \frac{R_1}{N}$	$r_2 + r_3 + \frac{R_1}{N^2}$
Figure 13(c)	As shown	R_3	$r_1 + r_2 + \frac{R_3}{N^2}$	$r_2 + \frac{R_3}{N}$	$r_4 + r_2 + \frac{R_3}{N}$	$r_2 + r_3 + R_3$
	Reversed	R_3	$r_1 + r_2 + \frac{R_3}{N^2}$	$r_2 - \frac{R_3}{N}$	$r_4 + r_2 - \frac{R_3}{N}$	$r_2 + r_3 + R_3$

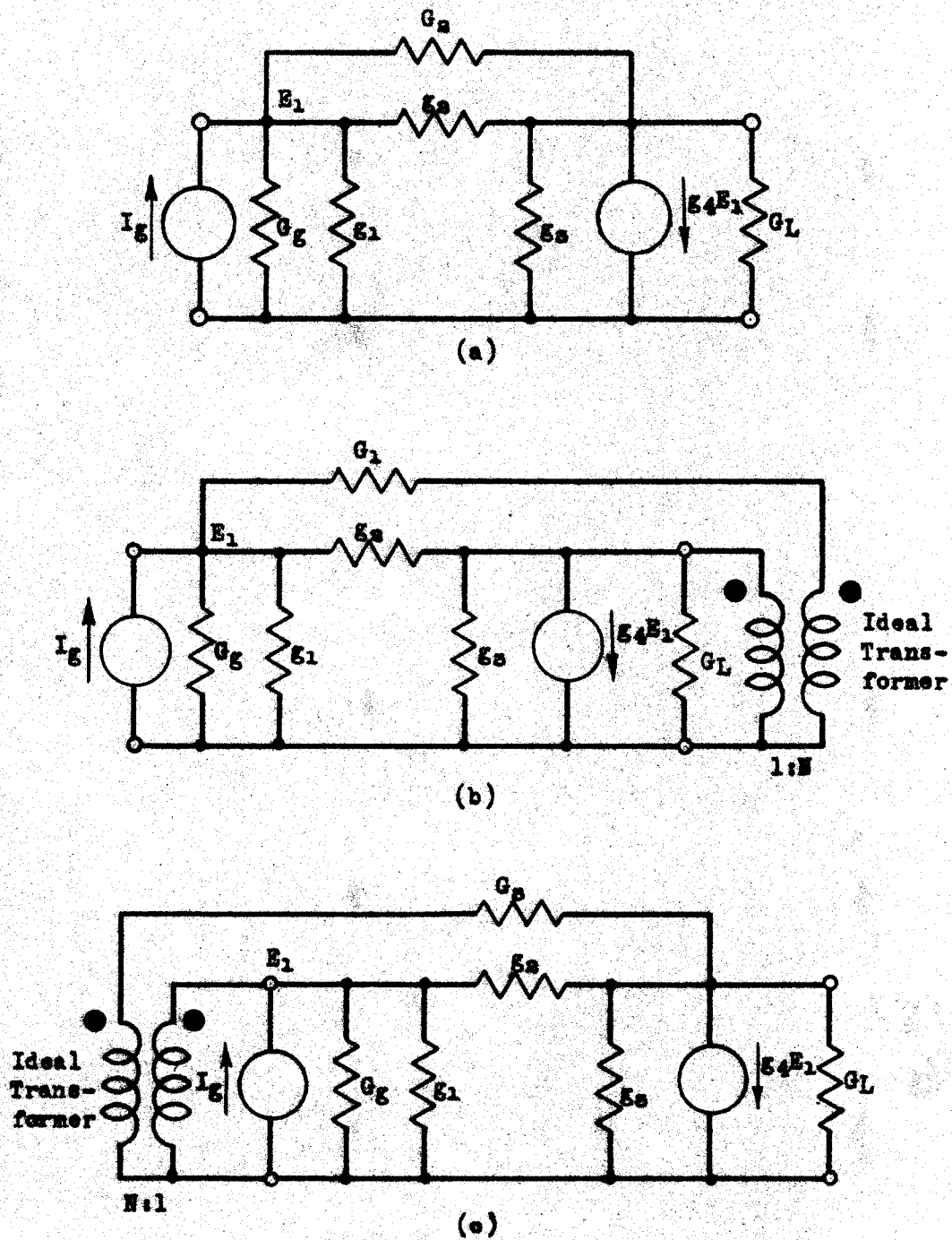


Figure 14. Parallel transistor feedback circuits

Table IV

Network Coefficients for Parallel Feedback Circuits

Circuit	Transformer Connection	Feedback Element	Network Coefficients			
			R_{11}	R_{12}	R_{21}	R_{22}
Figure 14(a)	---	G_2	$(\xi_1 + \xi_2 + G_2)$	$-(\xi_2 + G_2)$	$-(-\xi_4 + \xi_2 + G_2)$	$(\xi_2 + \xi_3 + G_2)$
Figure 14(b)	As shown	G_1	$(\xi_1 + \xi_2 + G_1)$	$-(\xi_2 + NG_1)$	$-(-\xi_4 + \xi_2 + NG_1)$	$(\xi_2 + \xi_3 + N^2 G_1)$
	Reverse	G_1	$(\xi_1 + \xi_2 + G_1)$	$-(\xi_2 - NG_1)$	$-(-\xi_4 + \xi_2 - NG_1)$	$(\xi_2 + \xi_3 + N^2 G_1)$
Figure 14(c)	As shown	G_3	$(\xi_1 + \xi_2 + N^2 G_3)$	$-(\xi_2 + NG_3)$	$-(-\xi_4 + \xi_2 + NG_3)$	$(\xi_2 + \xi_3 + G_3)$
	Reversed	G_3	$(\xi_1 + \xi_2 + N^2 G_3)$	$-(\xi_2 - NG_3)$	$-(-\xi_4 + \xi_2 - NG_3)$	$(\xi_2 + \xi_3 + G_3)$

are very similar in form to the network coefficients of the series feedback circuits. This is explained by the duality relationship which exists between the mesh- and nodal-derived equivalent circuit.

The most important feedback circuits of Figures 13 and 14 are the circuits not requiring transformers. Where transformers are used, the natural advantages which the transistor has in size, weight and cost are sacrificed. However, where these advantages are not important, transformers can be used for more versatile circuit design. Several types of transformers designed particularly for transistors have been made available¹.

B. Selection of Circuit Terminations

Impedance matching is one of the principal difficulties in designing transistor circuits. When dealing with vacuum tubes at low frequencies, the input impedance is usually considered infinite in grounded cathode or grounded plate operation. Voltage gain is considered the all-important factor because power gain is meaningless. In transistor circuits, the input impedance is not negligible and must be considered as a load for the preceding stage. This is

¹Standard Transformer Corporation. Stancoor transistor transformer data sheets. Form 462, 2-53 and Bulletin No. 463, 1-53. 3580 Elston Avenue, Chicago, Illinois. 1953.

similar to grounded grid operation in vacuum tubes. Power gain reappears in these cases as the most fundamental consideration of gain.

In designing transistor circuits, every effort should be made to match the transistor in its image impedances. This permits the maximum allowable power gain. Failure to recognize this may require additional stages in the over-all amplifier.

The use of transformers greatly simplifies the problem of matching. However, the additional weight, space, and cost is undesirable. This is especially true in miniaturized circuits for which the transistor is ideally suited. Stability may also be a problem when transformers are used.

In cases where transformers cannot be used, matched conditions are seldom obtained. Where only one stage is needed, it is sometimes possible to design the driving source and load to match the transistor. In cases where transistors are cascaded, the matching of source and load does not solve the problem. A mismatch still exists between stages. A better match can be obtained in cascaded circuits by alternating stages from one type connection to another. This will be discussed in Section V-B.

In the discussion which follows, we will be primarily concerned with matched conditions at the input and output of single-stage or cascaded transistor circuits.

C. Analysis of Circuits Containing Series or Parallel Feedback

1. The general transistor circuit

A systematic approach to the analysis of feedback and stability can best be made by first considering the general transistor circuits. General equations pertaining to these circuits can be derived and easily applied to specific cases.

Figure 12 shows the general mesh-derived equivalent circuit with terminations. R_g is the internal impedance of the driving source or the output impedance of the preceding stage. R_L is the load impedance or the input impedance of the succeeding stage. The mesh equations for the circuit are

$$E_g = (R_g + r_{11})I_1 + r_{12}I_2, \quad (33)$$

and

$$0 = r_{21}I_1 + (r_{22} + R_L)I_2. \quad (34)$$

Here r_{11} , r_{12} , r_{21} , r_{22} represent the network coefficients of the transistor when input and output terminals are shorted. This notation will be adhered to throughout.

The input and output impedances, measured at the transistor terminals, can be written as

$$R_{im} = \frac{\Delta_m}{\Delta_{m11}} \Bigg|_{R_g = 0} = \frac{r_{11}(r_{22} + R_L) - r_{12}r_{21}}{(r_{22} + R_L)} \quad (35)$$

and

$$R_{om} = \frac{\Delta_m}{\Delta_{m_{22}}} \bigg|_{R_L = 0} = \frac{(R_g + r_{11})r_{22} - r_{12}r_{21}}{(R_g + r_{11})}. \quad (36)$$

Here the subscript i or o designates input or output resistance and the subscript m designates the mesh-derived equivalent circuit. The symbol Δ_m represents the circuit determinant obtained from equations (33) and (34).

The voltage, current, and power gain of the transistor from input terminals to output terminals represented by A_E , A_I , and A_P respectively, are

$$A_{Em} = \frac{E_2}{E_1} = \frac{r_{21}R_L}{r_{11}(r_{22} + R_L) - r_{12}r_{21}}, \quad (37)$$

$$A_{Im} = -\frac{I_2}{I_1} = \frac{r_{21}}{r_{22} + R_L}, \quad (38)$$

and

$$A_{Pm} = A_{Em}A_{Im}. \quad (39)$$

These gain expressions must be properly interpreted. Equation (37) represents the ratio of output voltage to input voltage. Both voltages are measured at the terminals of the transistor. Equation (38) represents the ratio of output current flowing in the load to input current supplied by the source. Equation (39) is the product of these two and represents the ratio of power out of the transistor to the power supplied to the input terminals of the transistor.

In most cases, the return difference can be used as a measure of sensitivity. If gain is defined as in equations (37) or (38), then the sensitivity would be calculated on the basis that constant voltage or current was supplied at the input terminals of the transistor. This is the same as saying that the input impedance of the transistor does not change with changes in r_4 . This is not true. Therefore, as a more useful measure of sensitivity, the voltage or current gain will be defined as

$$A_{Em} \equiv \frac{E_2}{E_g}, \quad (40)$$

and

$$A_{Im} \equiv -\frac{I_2}{I_g}, \quad (41)$$

where E_g is the generator voltage of the source as shown in Figure 12 and I_g is the current of the current-source equivalent. In either case, R_g must be included in the circuit. This applies to the calculation of return difference and sensitivity only.

If the return difference for r_4 were used for each of the three connections of the transistor, the return difference would have a different meaning in each case. This is explained by the fact that r_m is not found in r_4 alone (see Table II). On the other hand, if return difference for r_m were used, then there would be only one meaning. As a measure of feedback, the return difference for r_m would indicate the amount feedback has reduced the gain in terms of the grounded base parameters. As a measure of sensitivity, the

return difference for r_m would indicate the effect that changes in r_m have in changing gain. The return difference for r_m will be used for all mesh-derived equivalent circuits. Both return difference and sensitivity will be derived separately for each connection.

The stability limit for the transistor is determined by the point at which the determinant goes to zero. For the general mesh-derived circuit this can be written as

$$\Delta_m = (R_g + r_{11})(r_{22} + R_L) - r_{12}r_{21} = 0. \quad (42)$$

All of the above expressions can also be used with added feedback. This is handled by using R_{11} , R_{12} , R_{21} , and R_{22} from Table III to replace r_{11} , r_{12} , r_{21} and r_{22} respectively.

For matched conditions, where $R_{im} = R_g$ and $R_{om} = R_L$, equations (35) through (39) are greatly simplified as follows:

$$R_{im} = r_{11}(1-a)^{1/2}, \quad (43)$$

$$R_{om} = r_{22}(1-a)^{1/2}, \quad (44)$$

$$A_{Em} = \frac{r_{21}/r_{11}}{1 + (1-a)^{1/2}}, \quad (45)$$

$$A_{Im} = \frac{r_{21}/r_{22}}{1 + (1-a)^{1/2}}, \quad (46)$$

and

$$A_{Pm} = A_{Em}A_{Im}. \quad (47)$$

In these expressions, a is defined by

$$a \equiv \frac{r_{12}r_{21}}{r_{11}r_{22}}. \quad (48)$$

For the circuit to be stable, a must be less than unity. The point

of instability for matched conditions occurs at

$$a = 1. \quad (49)$$

When added feedback is used, the above expressions for matched conditions are valid providing the network coefficients of Table III are used.

Figure 15 shows the general nodal-derived equivalent circuit with terminations. For ease of handling, the driving source is assumed to be a current source I_g shunted by the conductance G_g .

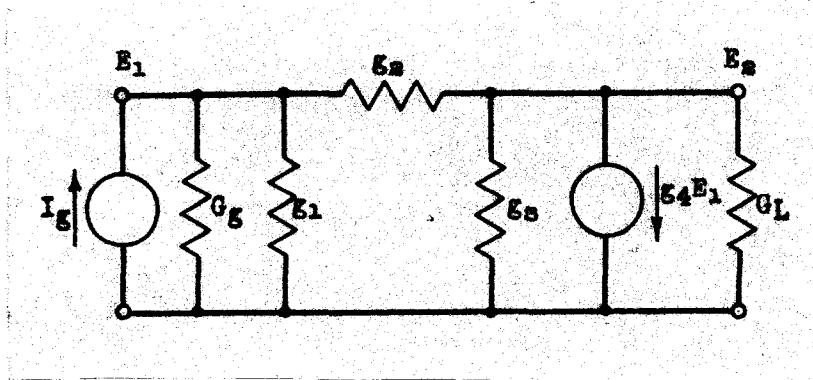


Figure 15. General nodal-derived equivalent circuit with terminations

The transistor load is represented by G_L .

For matched conditions, the equations for the nodal-derived equivalent circuit, analogous to equations (43) and (44) can be written as

$$G_{in} = \epsilon_{11}(1-b)^{1/2}, \quad (50)$$

and

$$G_{on} = \epsilon_{22}(1-b)^{1/2}, \quad (51)$$

where the i or o subscript designates input or output conductance and the n signifies the nodal-derived equivalent circuit.

The equations analogous to equations (45) through (47) are

$$A_{En} = \frac{E_2}{E_1} = \frac{-g_{21}/g_{22}}{1 + (1-b)^{1/2}}, \quad (52)$$

$$A_{In} = -\frac{I_2}{I_1} = \frac{-g_{21}/g_{11}}{1 + (1-b)^{1/2}} \quad (53)$$

and

$$A_{Pn} = A_{En}A_{In}. \quad (54)$$

In these expressions, b is defined as

$$b \equiv \frac{g_{12}g_{21}}{g_{11}g_{22}}. \quad (55)$$

For matched conditions, the circuit is stable when b is less than unity. The point of instability is then determined by the condition

$$b = 1. \quad (56)$$

If added feedback is to be used, the network coefficients G_{11} , G_{12} , G_{21} and G_{22} from Table IV must be substituted by g_{11} , g_{12} , g_{21} and g_{22} respectively for these relationships to remain valid.

It would again be desirable to use the return difference for r_m when calculating return difference for the nodal-derived equivalent circuit. However, due to the extreme complexities in computation, it is necessary to introduce a new parameter. The parameter chosen is $\frac{r_m}{|r|}$ which is the negative of g_4 for the grounded base connection.

The return difference for $\frac{r_m}{|r|}$ will be used for all of the nodal-derived equivalent circuits. Both return difference and sensitivity will be derived separately for each case that follows.

2. The grounded base transistor circuit

Without added feedback, the matched impedances for a typical junction transistor connected as grounded base can be calculated from equations (43) and (44) using values from Table II. They are

$$R_{imb} = 372(1-0.854)^{1/2} = 142 \text{ ohms}, \quad (57)$$

and

$$R_{omb} = 785,000(1-0.854)^{1/2} = 300,000 \text{ ohms}. \quad (58)$$

The subscript b has been added to indicate the grounded base.

The transistor gains for matched conditions, using equations (45), (46) and (47) are respectively

$$A_{Emb} = \frac{719,000/372}{1 + (1-0.854)^{1/2}} = 1400, \quad (59)$$

$$A_{Imb} = \frac{719,000/785,000}{1 + (1-0.854)^{1/2}} = 0.662, \quad (60)$$

and

$$A_{Pmb} = (1400)(0.662) = 924. \quad (61)$$

The return difference for r_m for the grounded base mesh-derived equivalent circuit is

$$F_{mb} = \frac{\Delta_{mb}}{\Delta_{mb}^0} = \frac{(R_g+r_{11})(r_{22}+R_L) - r_{12}r_{21}}{(R_g+r_{11})(r_{22}+R_L) - r_{12}^2}, \quad (62)$$

where Δ_{mb}^0 is the value of the circuit determinant where r_m is set to zero. With reference to Table II, a good approximation can be made by neglecting the second term in the denominator. Making this approximation, the return difference under matched conditions becomes

$$F_{mb} \cong \frac{2(1-ab)^{1/2}}{1 + (1-ab)^{1/2}} \quad (63)$$

Evaluating the return difference with the help of Table II gives

$$F_{mb} \cong \frac{2(1-0.854)^{1/2}}{1 + (1-0.854)^{1/2}} = 0.552 \quad (64)$$

The sensitivity for r_m in this case is

$$S_{mb} = \frac{1}{\frac{dA_{Emb}}{dr_m} \cdot \frac{r_m}{A_{Emb}}} = \left[\frac{r_{s1}}{r_{s1}-r_{1s}} \right] \left[\frac{(R_g+r_{11})(r_{ss}+R_L) - r_{1s}r_{s1}}{(R_g+r_{11})(r_{ss}+R_L)} \right] \quad (65)$$

Here, a good approximation is

$$\frac{r_{s1}}{r_{s1}-r_{1s}} \cong 1 \quad .$$

With this approximation, the sensitivity for matched conditions is

$$S_{mb} \cong \frac{2(1-ab)^{1/2}}{1 + (1-ab)^{1/2}} \quad (66)$$

which is the same as the expression obtained for return difference.

For the grounded base nodal-derived equivalent circuit without

added feedback, the return difference for $\frac{r_m}{|r|}$ is

$$F_{nb} = \frac{\Delta_{nb}}{\Delta_{nb}^0} = \frac{(G_g + \epsilon_{11})(\epsilon_{22} + G_L) - \epsilon_{12}\epsilon_{21}}{(G_g + \epsilon_{11})(\epsilon_{22} + G_L) - \epsilon_{12}^2}, \quad (67)$$

where Δ_{nb}^0 is the circuit determinant with $\frac{r_m}{|r|}$ set equal to zero.

As in the previous case, reference to Table II shows that the second term in the denominator can be neglected. Using this approximation, the return difference for matched conditions can be written

$$F_{nb} \cong \frac{2(1-b_b)^{1/2}}{1 + (1-b_b)^{1/2}}. \quad (68)$$

Without added feedback, this has the same value as equation (64).

The sensitivity for $\frac{r_m}{|r|}$ is

$$\begin{aligned} S_{nb} &= \frac{1}{\frac{dA_{Inb}}{d\left(\frac{r_m}{|r|}\right)} \cdot \frac{r_m}{|r|} A_{Inb}}, \\ &= \left[\frac{\epsilon_{21}}{\epsilon_{21} - \epsilon_{12}} \right] \left[\frac{(G_g + \epsilon_{11})(\epsilon_{22} + G_L) - \epsilon_{12}\epsilon_{21}}{(G_g + \epsilon_{11})(\epsilon_{22} + G_L)} \right]. \end{aligned} \quad (69)$$

Omitting the first factor, the sensitivity for matched conditions becomes

$$S_{nb} \cong \frac{2(1-b_b)^{1/2}}{1 + (1-b_b)^{1/2}}, \quad (70)$$

which is the same as equation (68).

Without added feedback, the circuit is stable since a_b or b_b is

always less than unity.

Figure 16 illustrates the effects of adding series feedback to the grounded base transistor circuit. Since series feedback is most easily handled by the mesh-derived equivalent circuit, the network coefficients R_{11} , R_{12} , R_{21} and R_{22} from Table III were used in place of r_{11} , r_{12} , r_{21} and r_{22} respectively. To simplify the problem, it was assumed that $N = 1$ for those circuits containing transformers. What this assumption does can be seen from the network coefficients of Table III and the circuits of Figure 13. With the transformers connected as shown, the network coefficients have the same form in all three cases. This represents added positive feedback and its effect upon the circuit characteristics is shown by the solid curves of Figure 16.

With the transformer reversed, the network coefficients of the circuits of Figures 13(b) and 13(c) have the same form. This represents added negative feedback and its effect upon the circuit characteristics are shown by the dashed curves of Figure 16.

It will be noted that both positive and negative feedback tend to raise the input impedance. This is as expected since R_{11} is increased sharply when either positive or negative feedback is added. R_{22} , on the other hand, is not greatly affected by feedback. The output impedance is therefore controlled primarily by the factor $(1-a_p)^{1/2}$. This explains the decrease in the output impedance when positive feedback is added.

Power gain decreases for negative feedback as would be expected.

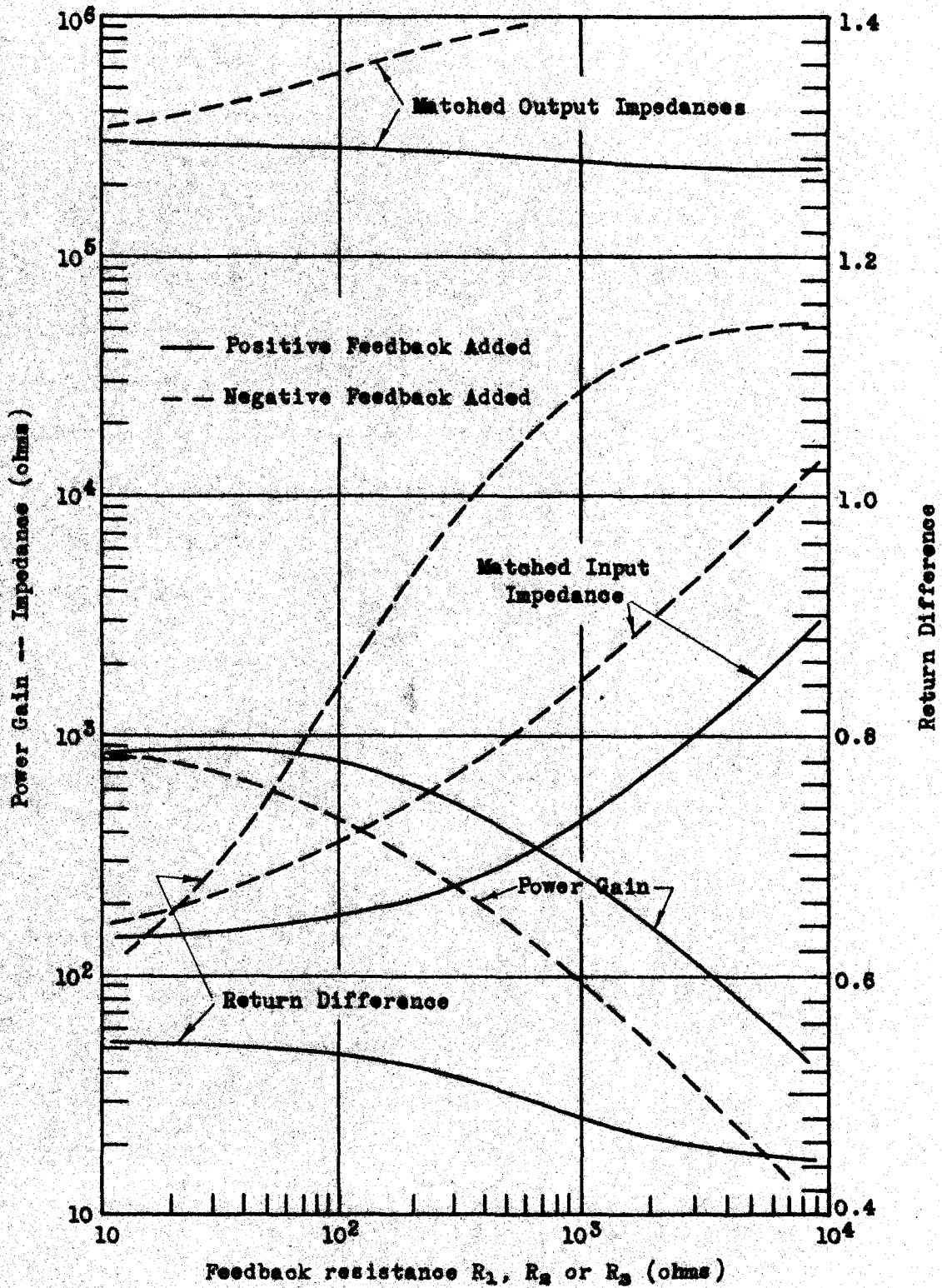


Figure 16. The typical grounded base junction transistor circuit with series feedback added

For positive feedback, the power gain increases only slightly at first and then also decreases. This can be attributed to the rapid increase in R_{11} which affects the voltage gain as shown by equation (45).

Equation (63) was used in calculating the return difference in Figure 16. The maximum error introduced by the approximation is less than one percent for the range of feedback resistance used. Likewise the maximum error introduced by the approximation for sensitivity (equation 66) is less than one and one-half percent. These errors are subtractive so that the curve for return difference represents sensitivity within one percent.

When series feedback is added, the circuit will be stable if

$$R_{11}R_{22} - R_{12}R_{21} > 0 . \quad (71)$$

This can easily be determined by substituting the network coefficients from Table III into equation (71). It can be shown that the circuit is stable when $N = 1$.

Parallel feedback is most easily handled by the nodal-derived equivalent circuit. Figure 17 shows the effect of adding parallel feedback to the grounded base connection. The curves are drawn for the typical junction transistor with $N = 1$. The solid curves represent the characteristics of all three parallel feedback circuits of Figure 14 when the added feedback is positive (transformers connected as shown). The dashed curves represent the characteristics of only those circuits containing transformers and with the added feedback negative (transformers reversed).

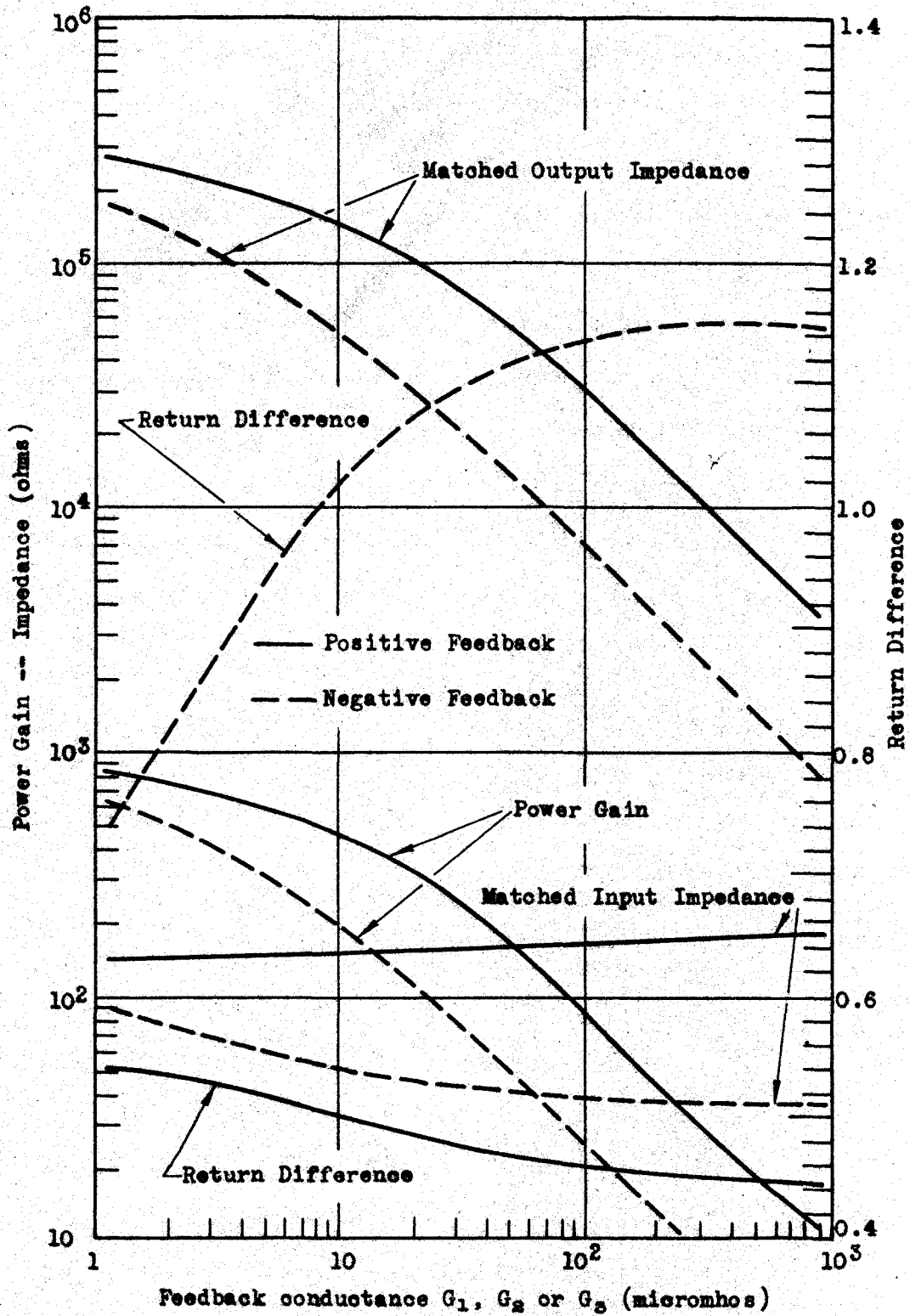


Figure 17. The typical grounded base junction transistor circuit with parallel feedback added

It will be noted that the characteristics of the parallel feedback circuits are considerably different from those of the series feedback circuits. The matched input impedances now remain somewhat constant, especially for positive feedback. This is explained by the fact that G_{11} changes only slightly with feedback and that the variation in input impedance is controlled primarily by $(1-b_b)^{1/2}$. On the other hand, G_{22} increases sharply with increasing feedback. We would expect the output impedance to decrease accordingly. This is shown by Figure 17.

Power gain decreases in both cases. This is due to the rapid increase in G_{22} as feedback is applied (see equation 54).

The approximations made in calculating return difference and sensitivity in equations (68) and (70) are good for all values of feedback conductance used. The curve for return difference represents sensitivity within two percent.

With $N = 1$, the circuit is stable for all values of feedback resistance used. If $N \neq 1$, stability is represented by the condition

$$G_{11}G_{22} - G_{12}G_{21} > 0, \quad (72)$$

where the network coefficients are those of Table IV.

3. The grounded emitter transistor circuit

The matched impedances for a typical junction transistor connected as grounded emitter without added feedback can be calculated

from equations (43) and (44). Substitution of values for the grounded emitter coefficients from Table II into these equations gives

$$R_{ime} = 487 \text{ ohms} , \quad (73)$$

and

$$R_{ome} = 86,500 \text{ ohms} . \quad (74)$$

The transistor gains from equations (45), (46) and (47) are

$$A_{E_{me}} = -837 , \quad (75)$$

$$A_{I_{me}} = -4.71 , \quad (76)$$

and

$$A_{P_{me}} = 3940 . \quad (77)$$

The return difference for r_m for the mesh-derived equivalent circuit is

$$F_{me} = \frac{\Delta_{me}}{\Delta_{me}^0} = \frac{(R_g + r_b + r_e)(r_o - r_m + r_e + R_L) - r_e(r_e - r_m)}{(R_g + r_b + r_e)(r_o + r_e + R_L) - r_e^2} . \quad (78)$$

Putting in the condition of matched terminating impedances and neglecting the second term in the denominator gives

$$F_{me} \approx \frac{2}{1 + \frac{r_{ss} + r_m}{r_{ss}(1 - a_e)^{1/2}}} . \quad (79)$$

Evaluating equation (79) with the help of Table II gives

$$F_{me} \approx 0.1985 . \quad (80)$$

The sensitivity for r_m can be written

$$S_{me} = \left[\frac{r_{s1}}{r_{s1} - r_{1s}} \right] \left[\frac{(R_g + r_b + r_e)(r_o - r_m + r_e + R_L) - r_e(r_e - r_m)}{(R_g + r_b + r_e)(r_o + R_L)} \right] . \quad (81)$$

Neglecting the first factor and putting in the condition of matched impedances gives

$$S_{me} \approx \frac{2}{1 + \frac{r_{aa} + r_m}{r_{aa}(1-a_e)^{1/2}}} \quad (82)$$

which is the same as equation (79).

The return difference and sensitivity for $\frac{r_m}{|r|}$ for the nodal-derived circuit can be shown to be

$$\begin{aligned} F_{ne} &\approx S_{ne} \\ &\approx \frac{2}{1 + \frac{\epsilon_{11} + \frac{r_m}{|r|}}{\epsilon_{11}(1-b_e)^{1/2}}} \quad (83) \end{aligned}$$

Without added feedback, the grounded emitter connection is stable.

The effects of adding series feedback to the typical grounded emitter circuit are shown by Figure 18. The dashed curves pertain to the circuit of Figure 13(a) and to the circuits of Figures 13(b) and 13(c) when $N = 1$ and when the transformers are connected as shown. This represents added negative feedback. The solid curves represent the circuits of Figures 13(b) and 13(c) when the transformers are reversed ($N = -1$). This represents added positive feedback.

With positive feedback added, the grounded-emitter circuit becomes unstable. The point of instability for the typical junction transistor with matched loads can be determined from equation (48) using the network coefficients from Table III.

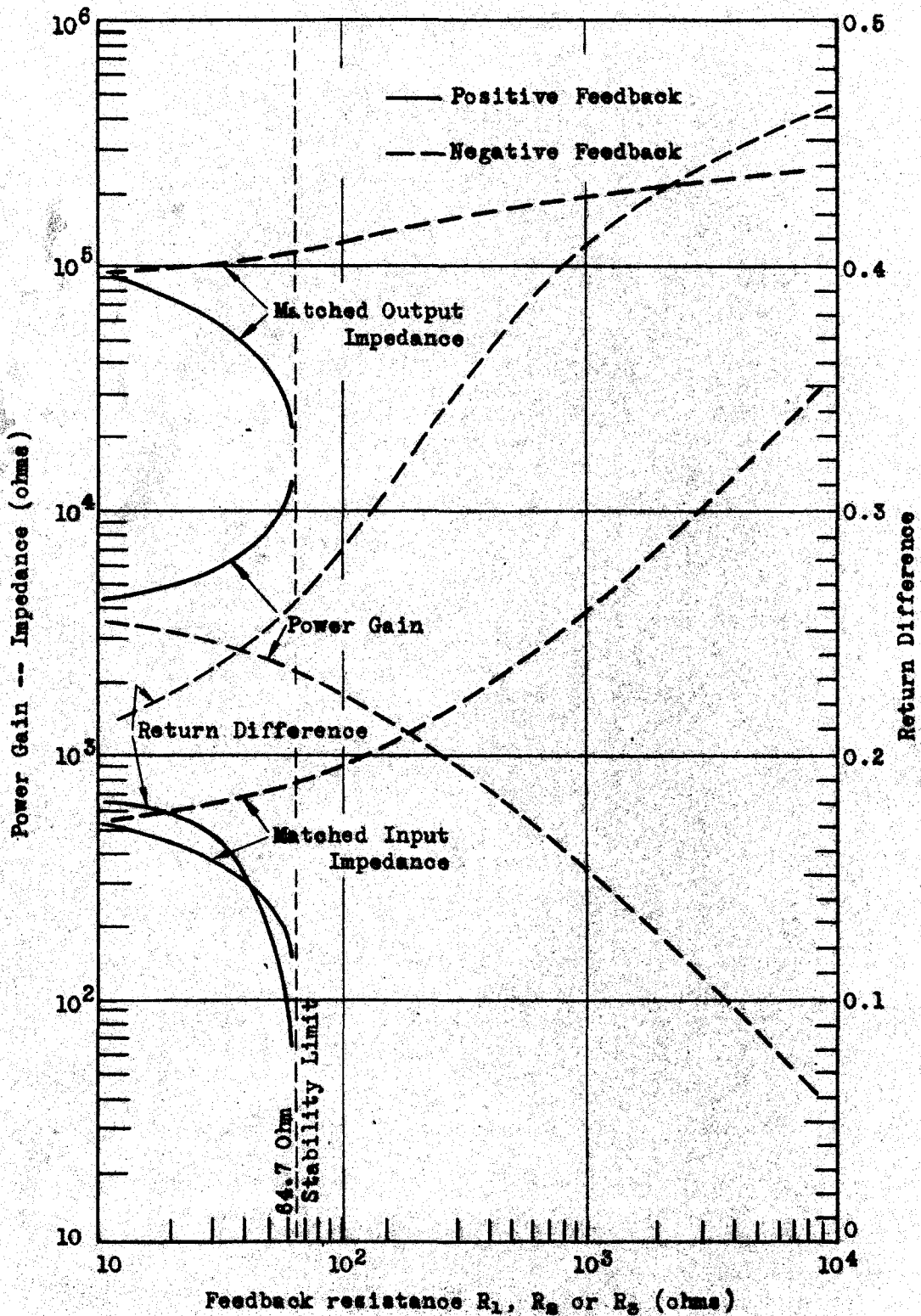


Figure 18. The typical grounded emitter junction transistor with series feedback added

$$a_{22} = \frac{(r_{12}-R_f)(r_{21}-R_f)}{(r_{11}+R_f)(r_{22}+R_f)} = 1 . \quad (84)$$

Here R_f represents R_1 , R_2 or R_3 depending upon which circuit is used. Solving equation (84) for R_f and evaluating by use of Table II gives

$$R_f = 64.7 \text{ ohms} . \quad (85)$$

With reference to Figure 18, the matched input impedance for the grounded emitter stage is generally higher than that of the grounded base stage; the matched output impedance is lower. This is highly desirable when matching problems are considered.

The grounded emitter connection produces higher power gain than any other connection. Without added feedback, this power gain approaches 4000. With negative feedback, the power gain decreases. With positive feedback, the power gain increases.

The plot of return difference of Figure 18 represents sensitivity within one percent. The curve shows that the circuit is inherently a positive feedback circuit when the circuit parameters are written in terms of the mesh-derived grounded base equivalent circuit parameters. The addition of negative feedback does not raise F above unity.

Figure 19 shows the characteristics of the typical grounded-emitter junction transistor circuit with parallel feedback added. The solid curves pertain to Figures 14(b) and 14(c) when the transformers are reversed for positive added feedback. The dashed curves represent the characteristics of all three circuits of Figure 14

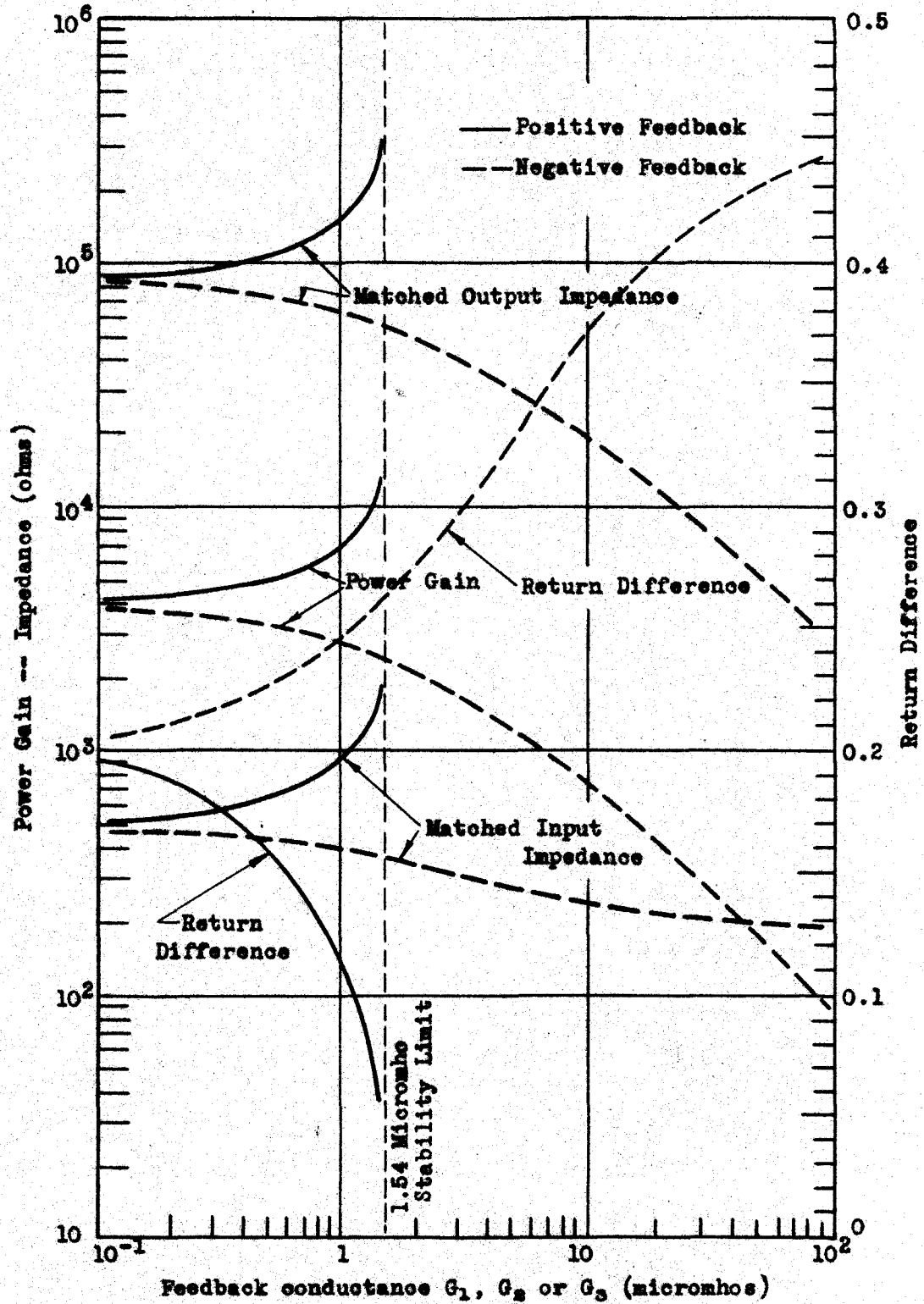


Figure 19. The typical grounded emitter junction transistor with parallel feedback added

when the transformers of Figures 14(b) and 14(c) are connected as shown. This represents negative added feedback. In all cases 1:1 transformers are assumed.

The circuit becomes unstable when the added feedback is positive. The point of instability can be found from equation (56) using the network coefficients of Table IV. For the typical transistor, this becomes

$$G_f = 1.54 \mu \text{ mhos} . \quad (86)$$

It will be noted that the matched impedances decrease for negative added feedback and increase for positive added feedback. This is opposite to the series feedback characteristics of Figure 18.

The plot of return difference for the parallel feedback circuit represents sensitivity within one percent.

4. The grounded collector transistor circuit

The matched impedances for the typical junction transistor connected as grounded collector can be calculated from equations (43) and (44). Without added feedback, they are

$$R_{inc} = 22,400 \text{ ohms} , \quad (87)$$

and

$$R_{omc} = 1860 \text{ ohms} . \quad (88)$$

The transistor gains, calculated from equations (45), (46) and (47) are

$$A_{Emc} = 0.973 , \quad (89)$$

$$A_{Imc} = 11.6 , \quad (90)$$

and

$$A_{Pmc} = 11.3 . \quad (91)$$

Equation (91) shows that the grounded collector stage does not produce as much power gain as the other two connections. However, equations (87) and (88) show that the circuit can be used to improve matching between two grounded emitter or two grounded base stages. This will be discussed in Section V-B.

The expressions for return difference and sensitivity for the grounded collector stage under matched conditions, can be shown to be

$$F_{mc} = \frac{2r_{11}r_{22} \left[1 - a_c + (1 - a_c)^{\frac{1}{2}} \right]}{r_m(r_{11} - r_{21}) + r_{11}r_m(1 - a_c)^{\frac{1}{2}} + r_{11}r_{22} \left[1 + (1 - a_c)^{\frac{1}{2}} \right]^2 + r_{21}(r_m - r_{21})} \quad (92)$$

and

$$S_{mc} = \frac{2r_{11}r_{22} \left[1 - a_c + (1 - a_c)^{\frac{1}{2}} \right]}{r_m(r_{11} - r_{12}) + r_{11}r_m(1 - a_c)^{\frac{1}{2}}} , \quad (93)$$

for the mesh-derived circuit. For the nodal-derived circuit, they are

$$F_{nc} = \frac{2g_{11}g_{22} \left[1 - b_c + (1 - b_c)^{\frac{1}{2}} \right]}{\frac{r_m}{|F|}(g_{22} + g_{21}) + g_{22} \frac{r_m}{|F|}(1 - b_c)^{\frac{1}{2}} + g_{11}g_{22} \left[1 + (1 - b_c)^{\frac{1}{2}} \right]^2 - g_{21} \left(\frac{r_m}{|F|} + g_{21} \right)} \quad (94)$$

and

$$S_{nc} = \frac{2\xi_{11}\xi_{22} \left[1 - b_c + (1 + b_c)^{1/2} \right]}{\frac{r_m}{|r|} (\xi_{22} + \xi_{21}) + \xi_{22} \frac{r_m}{|r|} (1 - b_c)^{1/2}} \quad (95)$$

Putting values into these equations shows that return difference is not a measure of sensitivity for the grounded collector. This is substantiated by the large amount of direct transmission present¹. If sensitivities are needed, they must be calculated from equations (93) or (95). For the typical transistor under matched conditions, the sensitivity for $\frac{r_m}{|r|}$ can be shown to be

$$S = 0.186 \quad (96)$$

Under matched conditions, all single stage grounded collector circuits of Figures 13 and 14 are stable when $N = 1$. If $N \neq 1$, the point of instability may be determined from equation (49) or equation (56) using the network coefficients of Table III or Table IV respectively.

D. Analysis of Circuits Containing Both Series and Parallel Feedback

All of the circuits discussed so far have been two-loop or two-node networks. When both series and parallel feedback are used on

¹Direct transmission here refers to the voltage or current gain of the circuit when r_m is made zero.

one transistor, a three-loop or three-node network results. It would be very desirable if the preceding methods and formulas could be applied to those circuits where both types of feedback are present. Two methods which make this possible will now be discussed.

Consider the equations for any mesh-derived transistor equivalent circuit given by equations (97) and (98).

$$E_1 = r_{11}I_1 + r_{12}I_2, \quad (97)$$

$$E_2 = r_{21}I_1 + r_{22}I_2. \quad (98)$$

The equations for the corresponding nodal-derived equivalent circuit are

$$I_1 = \frac{r_{22}}{|r|} E_1 - \frac{r_{12}}{|r|} E_2, \quad (99)$$

$$I_2 = -\frac{r_{21}}{|r|} E_1 + \frac{r_{11}}{|r|} E_2, \quad (100)$$

where

$$|r| = r_{11}r_{22} - r_{12}r_{21}. \quad (101)$$

If parallel feedback is added to the circuit as in Figure 13(a), equations (99) and (100) become

$$I_1 = \left[\frac{r_{22}}{|r|} + G_2 \right] E_1 - \left[\frac{r_{12}}{|r|} + G_2 \right] E_2, \quad (102)$$

$$I_2 = -\left[\frac{r_{21}}{|r|} + G_2 \right] E_1 + \left[\frac{r_{11}}{|r|} + G_2 \right] E_2. \quad (103)$$

Converting from the nodal-derived equivalent circuit back to a mesh-derived equivalent circuit, the corresponding equations can be

written as

$$E_1 = \left[\frac{r_{11} + G_2 |r|}{p} \right] I_1 + \left[\frac{r_{12} + G_2 |r|}{p} \right] I_2 , \quad (104)$$

$$E_2 = \left[\frac{r_{21} + G_2 |r|}{p} \right] I_1 + \left[\frac{r_{22} + G_2 |r|}{p} \right] I_2 , \quad (105)$$

where

$$p = 1 + G_2(r_{11} + r_{22} - r_{12} - r_{21}) . \quad (106)$$

The equivalent circuit corresponding to equations (104) and (105) is shown in Figure 20. The addition of parallel feedback to the mesh-derived equivalent circuit adds a resistance of $G_2 |r|$ ohms in series with the shunt leg and decreases all parameters (including the added resistance) by the factor p . Series feedback may now be added in the usual manner.

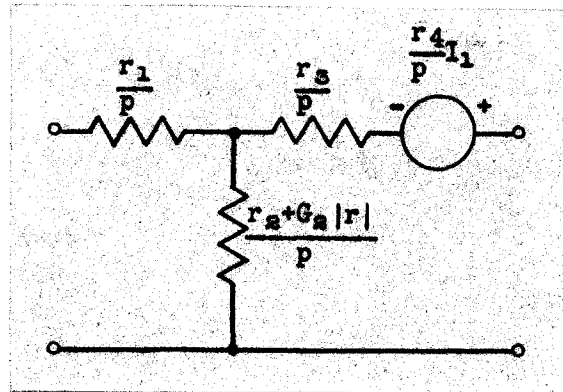


Figure 20. Mesh-derived equivalent circuit with parallel feedback

If a nodal derived equivalent circuit is desired, the circuit

of Figure 21 should be used. Here, the addition of series feedback to the nodal-derived equivalent circuit adds a conductance $R_2 |g|$ mhos

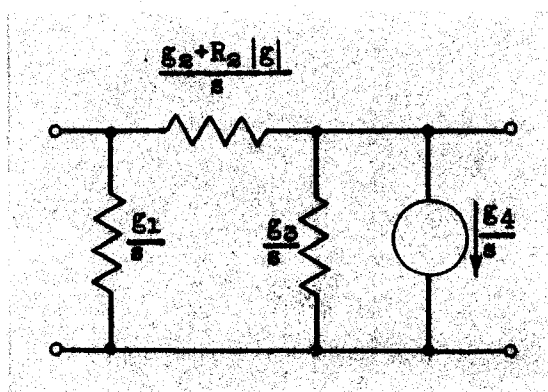


Figure 21. Nodal-derived equivalent circuit with series feedback

in parallel with the shunt branch and reduces all the parameters by the factor s where

$$|g| = g_{11}g_{22} - g_{12}g_{21}, \quad (107)$$

and

$$s = 1 + R_2(g_{11} + g_{22} - g_{12} - g_{21}). \quad (108)$$

Parallel feedback may now be added in the usual manner.

The above methods for handling circuits with both parallel and series feedback have been illustrated for circuits without transformers. This does not mean that transformers cannot be used. With proper interpretation, the above equivalent circuits can be modified to include transformers.

V. CASCADED TRANSISTOR CIRCUITS

A. Equivalent Circuits for Cascaded Transistors

In cases of no added feedback or where feedback is applied to individual stages, cascaded transistor circuits may be analyzed stage by stage. The analysis must begin with the last stage and proceed backwards until all stages have been treated.

In cases of added feedback where the feedback paths involve two or more stages, stage by stage analysis cannot be used. A method which can be used consists of deriving an equivalent circuit to replace two or more transistors. Feedback can then be applied to the new equivalent circuit in the usual way.

Consider two transistors connected in cascade. Let each transistor be represented by a mesh-derived equivalent circuit. If the input current and voltage to each transistor are considered to be the independent variables, the matrix equations for the first transistor can be written as

$$\begin{pmatrix} E_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} \frac{r'_{11}}{r'_{21}} & \frac{r'}{r'_{21}} \\ \frac{1}{r'_{21}} & \frac{r'_{22}}{r'_{21}} \end{pmatrix} \times \begin{pmatrix} E_j \\ -I_j \end{pmatrix}, \quad (109)$$

where

$$|r'| \equiv r'_{11}r'_{22} - r'_{12}r'_{21} \quad (110)$$

The equations for the second transistor can be written as

$$\begin{pmatrix} E_j \\ -I_j \end{pmatrix} = \begin{pmatrix} \frac{r''_{11}}{r''_{21}} & \frac{|r''|}{r''_{21}} \\ \frac{1}{r''_{21}} & \frac{r''_{22}}{r''_{21}} \end{pmatrix} \times \begin{pmatrix} E_2 \\ -I_2 \end{pmatrix} \quad (111)$$

where

$$|r''| \equiv r''_{11}r''_{22} - r''_{12}r''_{21} \quad (112)$$

Here, the common voltage and current at the junction between the transistors has been given the subscript j . The current I_j is assumed to flow into the first transistor.

Upon elimination of E_j and I_j by matrix methods, the following matrix is obtained:

$$\begin{pmatrix} E_1 \\ I_1 \end{pmatrix} = \frac{1}{r'_{21}r''_{21}} \begin{pmatrix} r'_{11}r''_{11} + |r'| & r'_{11}|r''| + r''_{22}|r'| \\ r''_{11} + r'_{22} & |r''| + r'_{22}r''_{22} \end{pmatrix} \times \begin{pmatrix} E_2 \\ -I_2 \end{pmatrix} \quad (113)$$

It is now possible to find a new mesh-derived equivalent circuit to replace the cascaded pair. Since the matrix for any one mesh-derived circuit takes the form of (109) or (111) we can write by inspection

$$\frac{r_{11}}{r_{21}} = \frac{r_{11}^i r_{11}^n + |r^i|}{r_{21}^i r_{21}^n}, \quad (114)$$

$$\frac{|r^i|}{r_{21}} = \frac{r_{11}^i |r^n| + r_{22}^n |r^i|}{r_{21}^i r_{21}^n}, \quad (115)$$

$$\frac{1}{r_{21}} = \frac{r_{11}^n + r_{22}^i}{r_{21}^i r_{21}^n}, \quad (116)$$

and

$$\frac{r_{22}}{r_{21}} = \frac{|r^n| + r_{22}^i r_{22}^n}{r_{21}^i r_{21}^n}, \quad (117)$$

where

$$|r| \equiv r_{11} r_{22} - r_{12} r_{21}. \quad (118)$$

Solving these expressions for r_{11} , r_{12} , r_{21} and r_{22} , the network coefficients for the new equivalent circuit are

$$r_{11} = \frac{r_{11}^i r_{11}^n + |r^i|}{r_{11}^n + r_{22}^i}, \quad (119)$$

$$r_{12} = \frac{r_{12}^i r_{12}^n}{r_{11}^n + r_{22}^i}, \quad (120)$$

$$r_{21} = \frac{r_{21}^i r_{21}^n}{r_{11}^n + r_{22}^i}, \quad (121)$$

and

$$r_{22} = \frac{|r^n| + r_{22}^i r_{22}^n}{r_{11}^n + r_{22}^i}. \quad (122)$$

If the first transistor is represented by a mesh-derived equivalent circuit and the second transistor is represented by the nodal-derived equivalent circuit, the mesh-derived equivalent circuit representing the pair is found from the following matrices:

$$\begin{pmatrix} E_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} \frac{r'_{11}}{r'_{21}} & \frac{|r'|}{r'_{21}} \\ 1 & \frac{r'_{22}}{r'_{21}} \end{pmatrix} \times \begin{pmatrix} E_j \\ -I_j \end{pmatrix}, \quad (123)$$

and

$$\begin{pmatrix} E_j \\ -I_j \end{pmatrix} = - \begin{pmatrix} \frac{\epsilon''_{22}}{\epsilon''_{21}} & \frac{1}{\epsilon''_{21}} \\ \frac{|\epsilon''|}{\epsilon''_{21}} & \frac{\epsilon''_{11}}{\epsilon''_{21}} \end{pmatrix} \times \begin{pmatrix} E_2 \\ -I_2 \end{pmatrix}. \quad (124)$$

Eliminating E_j and I_j and solving for the network coefficients

gives

$$r_{11} = \frac{r'_{11}\epsilon''_{22} + |r'| |\epsilon''|}{\epsilon''_{22} + |\epsilon''| r'_{22}}, \quad (125)$$

$$r_{12} = - \frac{r'_{12}\epsilon''_{12}}{\epsilon''_{22} + |\epsilon''| r'_{22}}, \quad (126)$$

$$r_{21} = - \frac{r'_{21}\epsilon''_{21}}{\epsilon''_{22} + |\epsilon''| r'_{22}}, \quad (127)$$

and

$$r_{22} = \frac{1 + r'_{22}\epsilon''_{11}}{\epsilon''_{22} + |\epsilon''| r'_{22}}. \quad (128)$$

In these expressions, we define

$$|r'| \equiv r'_{11}r'_{22} - r'_{12}r'_{21}, \quad (129)$$

Table V

Network Coefficients for Cascaded Pairs

Individual Transistor Representation Network Coefficients*

Mesh-derived resultant

1st Tran- sistor	2nd Tran- sistor	r_{11}	r_{12}	r_{21}	r_{22}
1 mesh	mesh	$\frac{r_{11}r_{11} + r_1}{r_{11} + r_1}$	$\frac{r_{12}r_{12}}{r_{11} + r_1}$	$\frac{r_{21}r_{21}}{r_{11} + r_1}$	$\frac{r_1 + r_{22}r_{22}}{r_{11} + r_1}$
2 mesh	nodal	$\frac{r_{11}r_{22} + r_1 \times \epsilon^n}{\epsilon^n + \epsilon^n r_{22}}$	$-\frac{r_{12}r_{21}}{\epsilon^n + \epsilon^n r_{22}}$	$-\frac{r_{21}r_{12}}{\epsilon^n + \epsilon^n r_{22}}$	$\frac{1 + r_{22}r_{21}}{\epsilon^n + \epsilon^n r_{22}}$
3 nodal	mesh	$\frac{r_{11}r_{22} + 1}{\epsilon^n + \epsilon^n r_{11}}$	$-\frac{r_{12}r_{21}}{\epsilon^n + \epsilon^n r_{11}}$	$-\frac{r_{21}r_{12}}{\epsilon^n + \epsilon^n r_{11}}$	$\frac{r_{22}r_{11} + r_1 \times \epsilon^n}{\epsilon^n + \epsilon^n r_{11}}$
4 nodal	nodal	$\frac{\epsilon^n r_{22} + \epsilon^n}{\epsilon^n + \epsilon^n + \epsilon^n r_{22}}$	$\frac{\epsilon^n r_{21}}{\epsilon^n + \epsilon^n + \epsilon^n r_{22}}$	$\frac{\epsilon^n r_{12}}{\epsilon^n + \epsilon^n + \epsilon^n r_{22}}$	$\frac{\epsilon^n r_{11} + \epsilon^n}{\epsilon^n + \epsilon^n + \epsilon^n r_{22}}$

*Single primes designate network coefficients of the first transistor. Double primes designate the network coefficients of the second transistor.

Table V (continued)

Individual Transistor Representation			Network Coefficients*			
Nodal-derived resultant			ξ_{11}	ξ_{12}	ξ_{21}	ξ_{22}
1st Transistor	2nd Transistor					
5	mesh	mesh	$\frac{r'_{22}r''_{22} + r'' }{ r'' r'_{11} + r' r''_{22}}$	$-\frac{r'_{12}r''_{12}}{ r'' r'_{11} + r' r''_{22}}$	$-\frac{r'_{21}r''_{21}}{ r'' r'_{11} + r' r''_{22}}$	$\frac{r'_{11}r''_{11} + r' }{ r'' r'_{11} + r' r''_{22}}$
6	mesh	nodal	$\frac{r'_{22}\xi''_{11} + 1}{r'_{11} + r' \xi''_{11}}$	$\frac{r'_{12}\xi''_{12}}{r'_{11} + r' \xi''_{11}}$	$\frac{r'_{21}\xi''_{21}}{r'_{11} + r' \xi''_{11}}$	$\frac{r'_{11}\xi''_{22} + r' \times \xi'' }{r'_{11} + r' \xi''_{11}}$
7	nodal	mesh	$\frac{r''_{22}\xi'_{11} + r'' \times \xi' }{r''_{22} + r'' \xi'_{22}}$	$\frac{r''_{12}\xi'_{12}}{r''_{22} + r'' \xi'_{22}}$	$\frac{r''_{21}\xi'_{21}}{r''_{22} + r'' \xi'_{22}}$	$\frac{r''_{11}\xi'_{22} + 1}{r''_{22} + r'' \xi'_{22}}$
8	nodal	nodal	$\frac{\xi'_{11}\xi''_{11} + \xi' }{\xi'_{11} + \xi'_{22}}$	$-\frac{\xi'_{12}\xi''_{12}}{\xi'_{11} + \xi'_{22}}$	$-\frac{\xi'_{21}\xi''_{21}}{\xi'_{11} + \xi'_{22}}$	$\frac{\xi'_{22}\xi''_{22} + \xi'' }{\xi'_{11} + \xi'_{22}}$

*Single primes designate network coefficients of the first transistor. Double primes designate the network coefficients of the second transistor.

for the first transistor and

$$|g''| \equiv \xi_{11}'' \xi_{22}'' - \xi_{12}'' \xi_{21}'' , \quad (130)$$

for the second transistor.

Table V summarizes the network coefficients for cascaded pairs including all possible combinations. Parameters for the first transistor are indicated by single primes. Parameters for the second transistor are indicated by double primes.

The process of combining transistors as summarized in Table V is actually circuit reduction of active networks. This process of reduction can be applied to any number of transistors in cascade; after two transistor circuits have been combined, the resultant circuit may be combined with the third and so on. The amount of work required is the only limitation.

The equivalent circuits corresponding to the network coefficients of Table V are not shown. They may be obtained by the methods outlined in Section III-A.

B. Analysis of Cascaded Circuits Containing Series or Parallel Feedback

Without the use of impedance matching devices, matched conditions are not obtained in cascaded transistor circuits. The most desirable connections, considering power gain only, are the grounded base and the grounded emitter connections. These connections have low input impedance and high output impedance. If transistor circuits of these

types are cascaded, much power gain will be lost due to mismatching.

On the other hand, the grounded collector stage has relatively high input impedance and low output impedance. Using a grounded collector stage alternately with the grounded base or grounded emitter stage produces a better match. However, the increase in power gain obtained from the grounded base or grounded emitter stage is offset by the low power gain of the grounded collector stage.

The above can be easily demonstrated for a two-stage transistor amplifier. Consider two grounded emitter stages connected in cascade. The network coefficients for the pair, obtained from equations (119) through (122) using values from Table II, are

$$r_{11} = \frac{(372)^2 + 42.2 \times 10^6}{66,400} = 638 \text{ ohms} , \quad (131)$$

$$r_{12} = \frac{(24.5)^2}{66,400} = 0.00904 \text{ ohms} , \quad (132)$$

$$r_{21} = \frac{(-719,000)^2}{66,400} = 7,800,000 \text{ ohms} , \quad (133)$$

and

$$r_{22} = \frac{42.2 \times 10^6 + 4.36 \times 10^9}{66,400} = 66,300 \text{ ohms} . \quad (134)$$

Equations (43) and (44) can now be used to find the image impedances of the cascaded pair. They are

$$R_{\text{image}} = 638 \text{ ohms} , \quad (135)$$

and

$$R_{omee} = 66,300 \text{ ohms} . \quad (136)$$

The power gain from equation (47) is

$$A_{Pmee} = 35,900 . \quad (137)$$

If the second stage were replaced by a grounded collector stage, equations (131) through (137) become

$$\begin{aligned} r_{11} &= 392 \text{ ohms} , \\ r_{12} &= 1.9 \text{ ohms} , \\ r_{21} &= -662,000 \text{ ohms} , \\ r_{22} &= 5,160 \text{ ohms} , \\ R_{imec} &= 499 \text{ ohms} , \\ R_{omec} &= 6,520 \text{ ohms} , \end{aligned} \quad (138)$$

and

$$A_{Pmec} = 42,000 \text{ ohms} .$$

If the first stage were a grounded collector stage and the second a grounded emitter stage, equations (131) through (137) become

$$\begin{aligned} r_{11} &= 5,040 \text{ ohms} , \\ r_{12} &= 24.4 \text{ ohms} , \\ r_{21} &= -8,500,000 \text{ ohms} , \\ r_{22} &= 66,500 \text{ ohms} , \\ R_{imee} &= 6,410 \text{ ohms} , \\ R_{omee} &= 84,500 \text{ ohms} , \end{aligned} \quad (139)$$

and

$$A_{Pmee} = 41,900 .$$

Equations (138) and (139) show that the grounded collector stage is very useful as a matching device. They show that higher gain may be obtained from a combination of grounded collector and grounded emitter stages than can be obtained from a circuit made up of all grounded emitter stages.

When feedback is added to cascaded transistor circuits, it is very easily handled by circuit reduction techniques. If the added feedback is applied to individual stages, as shown in Figures 13 and 14, then the corresponding network coefficients of Tables III and IV are used in place of the r 's and g 's of Table V. If the added feedback involves more than one stage, such as shown by Figures 22(a) and 22(b), the feedback elements R_2^i or G_2^i can be added after

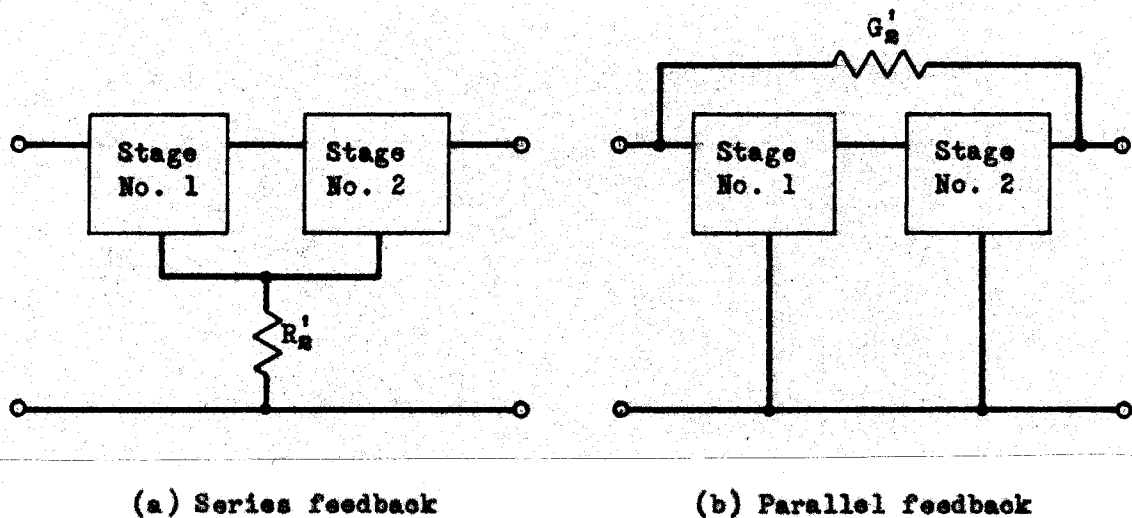


Figure 22. Cascaded transistor pair with added feedback

the circuit has been reduced. The addition of these feedback elements to the reduced circuit is identical to the addition of feedback to single stages in that R_2^i adds directly to r_2 and G_2^i adds directly to g_2 where r_2 and g_2 are parameters of the two-stage equivalent circuit (see Figures 2 and 3). The individual transistors of Figure 22 can be represented by either mesh- or nodal-derived equivalent circuits. However, for ease of handling, the resultant two-stage equivalent circuit must be a mesh-derived equivalent circuit for Figure 22(a) and a nodal-derived equivalent circuit for Figure 22(b). This is indicated by the type of feedback present.

A more complicated series feedback circuit, included merely for illustration, is shown in Figure 23. Here circuit reduction can be

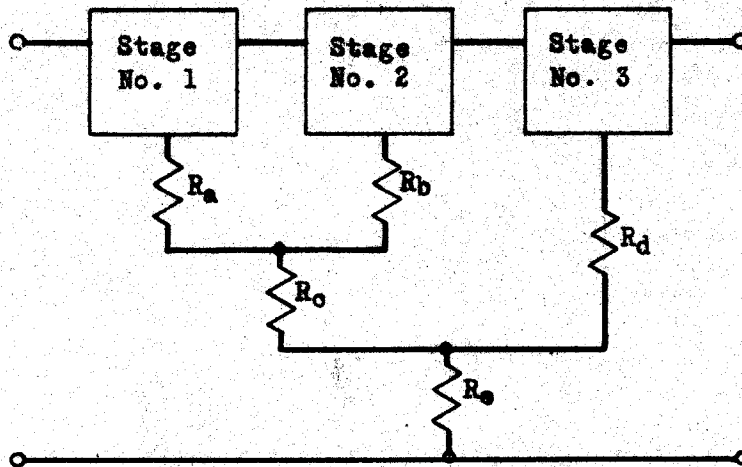


Figure 23. Three-stage transistor feedback circuit

applied to the first two stages with R_a and R_b included. With R_c added to the resultant two-stage equivalent circuit and with R_d added to the equivalent circuit of the last stage, the circuit can be reduced to one equivalent circuit for the three stages. Finally R_e can be added to the three-stage equivalent circuit to complete the problem.

Similarly, cascaded circuits with parallel feedback can be easily reduced with the use of nodal-derived equivalent circuits. No further explanation of this seems necessary.

The equations developed in Section IV-C for application to single-stage transistor circuits can be applied to cascaded transistor circuits without additional work. The equivalent circuit representing a cascaded transistor circuit is in no way different from the equivalent circuit of a single stage except in the value of the parameters. Using the network coefficients of the cascaded circuit, equations (43), (44), (50) and (51) will yield the matched input and output terminations for the cascaded circuit. Equation (47) or (54) will yield the power gain of the cascaded circuit. The equations for voltage gain, current gain and stability also apply.

C. Analysis of Cascaded Transistor Circuits Containing Both Series and Parallel Feedback

To illustrate the method of handling circuits containing both series and parallel feedback, Figure 24 is shown. Circuit reduction

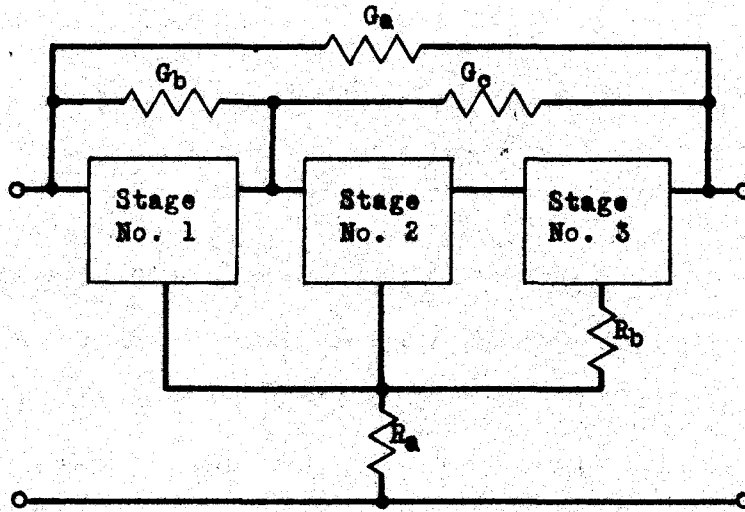


Figure 24. Cascaded transistor circuit with both series and parallel feedback

can be applied to this circuit in the following way.

Let the first stage be represented by a nodal-derived equivalent circuit. G_b may be added directly to this circuit. Let the second and third stages be represented by a mesh-derived equivalent circuit. R_b may be added directly to the third stage. The last two stages can now be combined by use of the relationships of Table V, line 5. This resultant two-stage equivalent circuit is of the nodal-derived type. Therefore G_c may be added directly. Now, the equivalent circuit of the first stage (with G_b included) can be combined with the two-stage equivalent circuit (with G_c and R_b included) resulting in a three-

stage equivalent circuit; it may be of either the mesh- or nodal-derived type.

If the three-stage equivalent circuit is obtained by use of the relationships of Table V, line 4, a mesh-derived circuit results. G_a may now be added to this circuit by methods outlined in Section IV-D. R_a may be added directly.

If the three-stage equivalent circuit is obtained by use of the relationships of Table V, line 8, a nodal-derived circuit results. Here, R_a may be added to the circuit by methods outlined in Section IV-D, and G_a may be added directly.

Regardless of what method is used, the circuit of Figure 24 can be reduced to one active network defined by four parameters. This is applicable to cascaded circuits in general.

VI. SUMMARY

A large selection of transistor equivalent circuits is available for use in transistor circuit analysis. In particular, those equivalent circuits which fit one of two general types are especially useful. These are the mesh-derived T and the nodal-derived π equivalent circuits. To reduce the number of equivalent circuit parameters and to greatly simplify the notation, all parameters can be written in terms of the grounded base mesh-derived equivalent circuit parameters: r_e , r_b , r_c , r_m .

The network coefficients pertaining to the nodal equations may be fairly easily measured by bridge methods; the mesh coefficients are comparatively difficult to measure with existing equipment. Any one of the three transistor connections may be used. Then, by means of mathematical relationships, the equivalent circuit parameters can be found from the network coefficients. Results obtained in the actual measurement of commercially available P-N-P junction transistors show that for identical bias conditions, the equivalent circuit parameters vary greatly between transistors.

The transistor is inherently a feedback device. Feedback may be added to single-stage transistor circuits with or without transformers. Weight, space and cost make transformers undesirable but their use permits more versatile circuit design. In either case, feedback may

be applied in one of two ways: series feedback which controls the shunt resistor of the T equivalent circuit and parallel feedback which controls the shunt resistor of the w equivalent circuit.

Circuits are best analyzed on the basis of matched conditions at the input and output terminals. General equations which hold for any type connection can be used in much of the analysis. A comprehensive study of series or parallel feedback on single-stage transistor circuits shows that matched input impedance, matched output impedance, voltage gain, current gain, power gain, return difference, sensitivity and stability are very much dependent upon feedback and can be controlled by feedback techniques. Each type connection is unique and must be studied separately. Stages with series and parallel feedback can also be easily handled.

Cascaded transistor circuits present no particular problem when circuit reduction of the active networks is employed. This method consists of obtaining a T or w equivalent circuit to replace two or more transistors. It is very versatile and can be used with series, parallel, or both series and parallel feedback. Feedback need not be limited to one stage but may involve any number of stages. Of most significance, after the cascaded circuit has been reduced, the analysis is identical to single stage analysis. All methods and formulas applicable to single stage circuits are applicable to reduced cascaded circuits.

VII. SELECTED REFERENCES

- Alexanderson, E. F. W. Control applications of the transistor. Proc. of the IRE. 40: 1508-1511. 1952.
- Anderson, A. Eugene. Transistors in switching circuits. Bell Sys. Tech. Jour. 31: 1207-1249. 1952.
- Angello, S. J. Semiconductor rectifiers. Elec. Eng. 68: 865-872. 1949.
- Banbury, P. C. Double surface lead sulphite transistor. Proc. Phys. Soc. (London). 65B: 236. 1952.
- _____ and Henisch, H. K. On the frequency response of PbS transistors. Proc. Phys. Soc. (London). 63B: 540-541. 1950.
- Bardeen, J. Surface states and rectification at a metal semiconductor contact. Phys. Rev. 71: 717-727. 1947.
- _____ . On the theory of the a-c impedance of a contact rectifier. Bell Sys. Tech. Jour. 28: 428-434. 1949.
- _____ . Theory of relation between hole concentration and characteristics of germanium point contacts. Bell Sys. Tech. Jour. 29: 469-495. 1950.
- _____ and Brattain, W. H. Transistor, a semiconductor triode. Phys. Rev. 74: 230-231. 1948.
- _____ and _____. Physical principles involved in transistor action. Phys. Rev. 75: 1208-1225. 1949.
- _____ and Pfann, W. G. Effects of electrical forming on the rectifying barriers of N- and P-germanium transistors. Phys. Rev. 77: 401-402. 1950.
- Becker, J. A. Photoeffects in semiconductors. Elec. Eng. 68: 937-942. 1949.
- _____ . Transistors. Elec. Eng. 69: 58-64. 1950.

- Becker, J. A. and Shive, J. N. The transistor, a new semiconductor amplifier. Elec. Eng. 68: 215-221. 1949.
- Becker, M. and Fan, H. Y. Photovoltaic effect of P-N junctions in germanium. Phys. Rev. 78: 301-302. 1950.
- Bell, D. A. Dual circuit of a feedback amplifier. Wireless Eng. 29: 40-43. Febr. 1952.
- Bode, Hendrik W. Network analysis and feedback amplifier design. 1st ed. N. Y., D. Van Nostrand Co., Inc. 1945.
- Brattain, W. H. and Bardeen, J. Nature of the forward current in germanium point contacts. Phys. Rev. 74: 231-232. 1948.
- _____ and _____. Surface properties of germanium. Bell Sys. Tech. Jour. 32: 1-41. 1953.
- _____ and Briggs, H. B. The optical constants of germanium in infra-red and visible. Phys. Rev. 75: 1705-1710. 1949.
- _____ and Shockley, W. Density of surface states on silicon deduced from contact potential measurements. Phys. Rev. 72: 345. 1947.
- Bray, Ralph. Dependence of resistivity of germanium on electric field. Phys. Rev. 76: 152-153. 1949.
- Briggs, H. B. Optical effects in bulk silicon and germanium. Phys. Rev. 77: 287. 1950.
- _____ . Infra-red absorption in silicon. Phys. Rev. 77: 727-728. 1950.
- Brown, C. B. Magnetically biased transistors. Phys. Rev. 76: 1736-1737. 1949.
- _____ . High frequency operation of transistors. Electronics. 23: 81-83. July 1950.
- Bryan, G. W., Jr. Application of transistors to high-voltage low-current supplies. Proc. of the IRE. 40: 1521-1523. 1952.

- Caruthers, R. S. Some system applications of transistor amplifiers. In Bell Telephone Laboratories, Inc. The transistor: selected reference material on characteristics and applications. p. 335-376. New York. The Laboratories. 1951.
- _____ . Some experimental and practical applications of transistor oscillators. In Bell Telephone Laboratories, Inc. The transistor: selected reference material on characteristics and applications. p. 397-414. New York. The Laboratories. 1951.
- Coblentz, Abraham and Owens, Harry L. Variation of transistor parameters with temperature. Proc. of the IRE. 40: 1472-1476. 1952.
- _____ and _____. Transistors: theory and application. Electronics. 26: 98-102. Mar. 1953.
- _____ and _____. Energy levels in transistor electronics. Electronics. 26: 138-141. Apr. 1953.
- _____ and _____. Physical properties of electrons in solids. Electronics. 26: 162-165. May 1953.
- _____ and _____. Transistor action in germanium and silicon. Electronics. 26: 166-171. June 1953.
- _____ and _____. Point-contact transistor operation. Electronics. 26: 158-163. July 1953.
- _____ and _____. Operation of junction transistors. Electronics. 26: 156-161. Aug. 1953.
- _____ and _____. Equivalent transistor circuits and equations. Electronics. 26: 156-161. Sept. 1953.
- Conwell, Esther M. Properties of silicon and germanium. Proc. of the IRE. 40: 1327-1337. 1952.
- _____ . High field mobility in germanium with impurity scattering dominant. Phys. Rev. 90: 769-772. 1953.
- _____ and Weisskopf, V. F. Theory of impurity scattering in semiconductors. Phys. Rev. 77: 388-390. 1950.
- Dacey, G. C. Space-charge limited hole current in germanium. Phys. Rev. 90: 759-763. 1953.

- Dacey, G. C. and Ross, I. M. Unipolar "field-effect" transistor. Proc. of the IRE. 41: 970-979. 1953.
- Danko, S. F. and Gerhold, R. A. Printed circuitry for transistors. Proc. of the IRE. 40: 1524-1528. 1952.
- Dunlap, W. Crawford, Jr. Important new semiconductor. G. E. Rev. 52: 9-17. Febr. 1949.
- Early, J. M. Effects of space-charge layer widening in junction transistors. Proc. of the IRE. 40: 1401-1406. 1952.
- Ebers, J. J. Four-terminal P-N-P-N transistors. Proc. of the IRE. 40: 1361-1364. 1952.
- Epstein, Gerald S., Bush, John A. and Shelhorn, Boyd. Transistorizing communications equipment. Electronics. 25: 98-102. May 1952.
- Fahnestock, James D. Experiments illustrate transistor applications. Electronics. 26: 112-113. Mar. 1953.
- _____. Transistorized hearing aids. Electronics. 26: 154-155. Apr. 1953.
- Fan, H. Y. Temperature dependence of the energy gap in semiconductors. Phys. Rev. 82: 900-905. 1951.
- Farley, B. G. Dynamics of transistor negative-resistance circuits. Proc. of the IRE. 40: 1497-1508. 1952.
- Fink, D. G. and Jurgen, R. K. The junction transistor. Electronics. 24: 82-85. Nov. 1951.
- _____. and Rocket, F. H. The transistor, a crystal triode. Electronics. 21: 68-71. Sept. 1948.
- Fuller, Calvin S. Diffusion of donor and acceptor elements into germanium. Phys. Rev. 86: 136-137. 1952.
- _____. and Struthers, J. D. Copper as an acceptor element in germanium. Phys. Rev. 87: 526-527. 1952.
- _____. Theuerer, H. C. and van Roesbroeck, W. Properties of thermally produced acceptors in germanium. Phys. Rev. 85: 678-679. 1952.

- Gebbie, H. A., Banbury, P. C. and Hogarth, C. A. Crystal diode and triode action in lead sulphide. Proc. Phys. Soc. (London). 63B: 371. 1950.
- General Radio Company. Operating instructions for type 561-D vacuum-tube bridge. 275 Massachusetts Avenue, Cambridge 39, Massachusetts. Jan. 1953.
- Giacoletto, L. J. Junction transistor characteristics at low and medium frequencies. Nat. Electronics Conf. Proc. 8: 321-329. 1952.
- _____. Junction transistor equivalent circuits and vacuum-tube analogy. Proc. of the IRE. 40: 1490-1493. 1952.
- _____. Terminology and equations for linear active four-terminal networks including transistors. RCA Review. 14: 28-46. 1953.
- _____. Equipments for measuring junction transistor admittance parameters for a wide range of frequencies. RCA Review. 14: 269-296. 1953.
- Golden, N. and Nielsen, R. Oscilloscopic display of transistor static electrical characteristics. Proc. of the IRE. 40: 1437-1439. 1952.
- Goucher, F. S. Photon yield of electron-hole pairs in germanium. Phys. Rev. 78: 816. 1950.
- _____. Measurement of hole diffusion in N-type germanium. Phys. Rev. 81: 475. 1951.
- _____, Pearson, G. L., Sparks, M., Teal, G. K. and Shockley, W. Theory and experiment for a germanium P-N junction. Phys. Rev. 81: 637-638. 1951.
- _____ and Prince, M. B. Interpretation of α -values in P-N junction transistors. Phys. Rev. 89: 651-653. 1953.
- Granville, J. W. and Bardsley, W. Some properties of silicon point-contact transistors. Proc. Phys. Soc. (London). 66B: 429. 1953.
- Guillemin, Ernst A. Communication networks, Vol. II. 1st ed. N. Y. John Wiley and Sons, Inc. 1935.
- Gunn, J. B. Radiative transitions in germanium. Proc. Phys. Soc. (London). 66B: 330-331. 1953.

- Haegle, R. W. Crystal tetrode mixer. *Electronics*. 22: 80-81. Oct. 1949.
- Hall, R. N. Electron-hole recombination in germanium. *Phys. Rev.* 87: 387. 1952.
- _____ . Power rectifiers and transistors. *Proc. of the IRE.* 40: 1512-1518. 1952.
- _____ and Dunlap, W. C. P-N junctions prepared by impurity diffusion. *Phys. Rev.* 80: 467-468. 1950.
- Haynes, J. R. and Shockley, W. Investigation of hole injection in transistor action. *Phys. Rev.* 75: 691. 1949.
- _____ and _____. The mobility and life of injected holes and electrons in germanium. *Phys. Rev.* 8: 835-843. 1951.
- Herring, Conyers. Theory of transient phenomena in the transport of holes in an excess semiconductor. *Bell Sys. Tech. Jour.* 28: 401-427. 1949.
- Hunter, Floyd P. Graphical analysis of transistor characteristics. *Proc. of the IRE.* 38: 1387-1391. 1950.
- Hunter, L. P. Physical interpretation of type A transistor characteristics. *Phys. Rev.* 77: 558-559. 1950.
- _____ and Brown, R. E. Production tester for transistors. *Electronics*. 23: 96-99. Oct. 1950.
- Kelley, Mervin J. The first five years of the transistor. *Bell Tele. Mag.* 32: 73-86. 1953.
- Keonjian, E. and Schaffner, J. S. An experimental investigation of transistor noise. *Proc. of the IRE.* 40: 1456-1460. 1952.
- _____ and _____. Noise in transistor amplifiers. *Electronics*. 26: 104-107. Febr. 1953.
- Klohr, Carl N. and Hunter, L. P. Measurement of semiconductor impurity content. *Phys. Rev.* 81: 1059-1060. 1951.
- Knight, Geoffrey, Jr., Johnson, Richard A. and Holt, Roland B. Measurement of the small signal parameters of transistors. *Proc. of the IRE.* 41: 983-989. 1953.

- Koch, W. E. and Wallace, R. L., Jr. The coaxial transistor. *Elec. Eng.* 68: 222-223. 1949.
- Kurshaw, J., Lohman, R. D. and Herzog, G. B. Cathode-ray tube plots transistor curves. *Electronics*. 26: 122-127. Febr. 1953.
- Law, R. R., Mueller, C. W., Pankove, J. I. and Armstrong, L. D. A developmental germanium P-N-P junction transistor. *Proc. of the IRE*. 40: 1352-1357. 1952.
- Lehan, Frank W. Transistor oscillator for telemetering. *Electronics*. 22: 90-91. Aug. 1949.
- Lehovee, K. On hole current in germanium transistors. *Phys. Rev.* 75: 1100-1101. 1949.
- _____ . Testing transistors. *Electronics*. 22: 88-89. June 1949.
- _____ . New photoelectric devices utilizing carrier injection. *Proc. of the IRE*. 40: 1407-1409. 1952.
- Lindberg, Olof. Hall effect. *Proc. of the IRE*. 40: 1414-1419. 1952.
- Linvill, J. G. Transistor negative-impedance converters. *Proc. of the IRE*. 41: 725-729. 1953.
- Lohman, Robert D. Complementary symmetry transistor circuits. *Electronics*. 26: 140-143. Sept. 1953.
- McAfee, K. B., Ryder, E. J., Shockley, W. and Sparks, M. Observations of zener currents in germanium P-N junctions. *Phys. Rev.* 83: 650-651. 1951.
- _____ , Shockley, W. and Sparks, M. Measurement of diffusion in semiconductors by a capacitance method. *Phys. Rev.* 86: 137-138. 1952.
- McKay, Kenneth G. The N-P-N junction as a model for secondary photoconductivity. *Phys. Rev.* 84: 833-835. 1951.
- Merrithew, R. F. Noise analyzer for transistor production. *Electronics*. 26: 136-137. May 1953.
- Montgomery, H. C. Background noise in transistors. *Bell Lab. Rec.* 28: 400-403. 1950.

- Montgomery, H. C. Electrical noise in semiconductors. Bell Sys. Tech. Jour. 31: 950-975. 1952.
- _____. Transistor noise in circuit applications. Proc. of the IRE. 40: 1461-1471. 1952.
- Morton, J. A. Present status of transistor development. Proc. of the IRE. 40: 1314-1326. 1952.
- Navon, D., Bray, R. and Fan, H. Y. Lifetime of injected carriers in germanium. Bull. Amer. Phys. Soc. 27: 14-15. June 30, 1952.
- O'Neill, B. J. and Gutterman, Arthur. Methods and equipment for transistor testing. Electronics. 26: 172-175. July 1953.
- Pearson, G. L. The physics of electronic semiconductors. Tran. of AIEE. 36: 209-214. 1947.
- _____. and Bardeen, J. Electrical properties of pure silicon and silicon alloys containing boron and phosphorous. Phys. Rev. 75: 865-883. 1949.
- _____. , Read, W. T. and Shockley, W. Probing the space charge layer in a P-N junction. Phys. Rev. 85: 1055-1057. 1952.
- _____. and Sawyer, B. Silicon P-N junction alloy diodes. Proc. of the IRE. 40: 1348-1352. 1952.
- Peterson, L. C. Equivalent circuits of linear active four-terminal networks. Bell Sys. Tech. Jour. 27: 593-622. 1948.
- Petriz, Richard L. On the theory of noise in P-N junctions and related devices. Proc. of the IRE. 40: 1440-1456. 1952.
- Pfann, W. G. and Scaff, J. H. P-germanium transistor. Proc. of the IRE. 38: 1151-1154. 1950.
- Pietenpol, W. J. P-N junction rectifier and photo-cell. Phys. Rev. 82: 120-121. 1951.
- Pritchard, R. L. Frequency variations of current-amplification factor for junction transistors. Proc. of the IRE. 40: 1476-1481. 1952.

- Raisbeck, G. A new high-efficiency linear amplifier using a transistor and a vacuum tube. In Bell Telephone Laboratories, Inc. The transistor: selected reference material on characteristics and applications. p. 377-387. New York. The Laboratories. 1951.
- _____ . A high-efficiency untuned amplifier. In Bell Telephone Laboratories, Inc. The transistor: selected reference material on characteristics and applications. p. 389-396. New York. The Laboratories. 1951.
- _____ . Transistor circuit design. Electronics. 24: 128-134. Dec. 1951.
- Raytheon Manufacturing Company. Germanium transistor data sheets No. CS-2881 and No. CS-2884. 55 Chapel Street, Newton 58, Massachusetts. Nov. 1952.
- Read, W. T. and Shockley, W. Dislocation models of crystal grain boundaries. Phys. Rev. 78: 275-289. 1950.
- Rose, G. M. and Slade, B. N. Transistors oscillate at 300 MC. Electronics. 25: 116-118. Nov. 1952.
- Roth, Louise and Taylor, W. E. Preparation of germanium single crystals. Proc. of the IRE. 40: 1338-1341. 1952.
- Rouault, C. L. and Hall, G. N. A high voltage, medium-power rectifier. Proc. of the IRE. 40: 1519-1521. 1952.
- Ryder, E. J. Mobility of holes and electrons in high electric fields. Phys. Rev. 90: 766-769. 1953.
- _____ and Shockley, W. Interpretation of dependence of resistivity of germanium on electric field. Phys. Rev. 75: 310. 1949.
- _____ and _____. Mobilities of electrons in high electric fields. Phys. Rev. 81: 139-140. 1951.
- Ryder, R. M. The type A transistor. Bell Lab. Rec. 27: 89-93. 1949.
- _____ . Aging and temperature response of point-contact transistors. In Bell Telephone Laboratories, Inc. The transistor: selected reference material on characteristics and applications. p. 123-125. New York. The Laboratories. 1951.

- Ryder, R. M. and Kirchner, R. J. Some circuit aspects of the transistor. Bell Sys. Tech. Jour. 28: 367-401. 1949.
- Saby, John S. Fused impurity P-N-P junction transistor. Proc. of the IRE. 40: 1358-1360. 1952.
- Schultheiss, P. M. and Reich, H. J. Some transistor trigger circuits. Proc. of the IRE. 39: 627-632. 1951.
- Scott, T. R. Crystal triodes. Elec. Comm. 28: 195-208. 1951.
- Shea, Richard F. Transistor operation: stabilization of operating points. Proc. of the IRE. 40: 1435-1437. 1952.
- _____. Principles of transistor circuits. 1st ed. N. Y. John Wiley and Sons, Inc. 1953.
- Shekel, Jacob. Matrix representation of transistor circuits. Proc. of the IRE. 40: 1493-1497. 1952.
- Shive, John N. The double surface transistor. Phys. Rev. 75: 689-690. 1949.
- _____. The phototransistor. Bell Lab. Rec. 28: 337-342. 1950.
- _____. Properties of the M-1740 P-N junction transistor. Proc. of the IRE. 40: 1410-1413. 1952.
- Shockley, W. The theory of P-N junctions in semiconductors and P-N junction transistors. Bell Sys. Tech. Jour. 28: 435-489. 1949.
- _____. Electrons and holes in semiconductors. 1st ed. N. Y., D. Van Nostrand Co., Inc. 1950.
- _____. Energy band structures in semiconductors. Phys. Rev. 78: 173. 1950.
- _____. Hot electrons in germanium and ohm's law. Bell Sys. Tech. Jour. 30: 990-1034. 1951.
- _____. Transistor electronics: imperfections, unipolar and analog transistors. Proc. of the IRE. 40: 1289-1313. 1952.
- _____. A unipolar "field-effect" transistor. Proc. of the IRE. 40: 1365-1376. 1952.

- Shockley, W. and Bardeen, J. Energy bands and mobilities in monatomic semiconductors. *Phys. Rev.* 77: 407-408. 1950.
- _____ and Pearson, G. L. Modulation of thin conductance films of semi-conductors by surface charges. *Phys. Rev.* 74: 232-233. 1948.
- _____, _____ and Haynes, J. R. Hole injection in germanium, quantitative studies and filamentary transistors. *Bell Sys. Tech. Jour.* 28: 344-366. 1949.
- _____ and Prim, R. C. Space-charge limited emission in semiconductors. *Phys. Rev.* 90: 753-758. 1953.
- _____ and Read, W. T., Jr. Statistics of recombinations of holes and electrons. *Phys. Rec.* 87: 835-842. 1952.
- _____, Sparks, M. and Teal, G. K. The P-N junction transistors. *Phys. Rev.* 83: 151-162. 1951.
- Sittner, W. R. Current multiplication in the type A transistor. *Proc. of the IRE.* 40: 448-454. 1952.
- Skalnik, J. G., Reich, H. J., Gibson, J. E. and Flynn, T. Auxiliary current alters transistor characteristics. *Electronics.* 24: 142, 228, 232, 236. Sept. 1951.
- Slade, B. N. A high-performance transistor with wide spacing between contacts. *RCA Review.* 11: 517-526. 1950.
- _____ . The control of frequency response and stability of point-contact transistors. *Proc. of the IRE.* 40: 1382-1384. 1952.
- _____ . Factors in the design of point-contact transistors. *RCA Review.* 14: 17-27. 1953.
- Slichter, W. P. and Kolb, E. D. Impurity effects in the thermal conversion of germanium. *Phys. Rev.* 87: 527-528. 1952.
- Spencer, Richard H. Transistor-controlled magnetic amplifier. *Electronics.* 26: 136-140. Aug. 1953.
- Standard Transformer Corporation. Stancor transistor transformer data sheets. Form 462, 2-53 and Bulletin No. 463, 1-53. 3580 Elston Avenue, Chicago, Illinois. 1953.
- Stansel, F. R. Transistor equations. *Electronics.* 26: 156, 158. Mar. 1953.

- Steele, Earl L. Theory of alpha for P-N-P diffused junction transistors. Proc. of the IRE. 40: 1424-1428. 1952.
- Stuetzer, O. M. Crystal amplifier with high input impedance. Proc. of the IRE. 38: 868-871. 1950.
- _____. Transistors in airborne equipment. Proc. of the IRE. 40: 1529-1530. 1952.
- Suhl, H. and Shockley, W. Concentrating holes and electrons by magnetic fields. Phys. Rev. 75: 1617-1618. 1949.
- Sulzer, Peter G. Junction transistor circuit applications. Electronics. 26: 170-173. Aug. 1953.
- _____. Low-distortion transistor audio oscillator. Electronics. 26: 171-173. Sept. 1953.
- Sziklai, George Clifford. Symmetrical properties of transistors and their applications. Proc. of the IRE. 41: 717-724. 1953.
- _____. Lohman, R. D. and Herzog, G. B. A study of transistor circuits for television. Proc. of the IRE. 41: 708-717. 1953.
- Teal, G. K. and Little, J. B. Growth of germanium single crystals. Phys. Rev. 78: 647. 1950.
- _____. Sparks, M. and Buehler, E. Growth of germanium single crystals containing P-N junctions. Phys. Rev. 81: 637. 1951.
- Thomas, D. E. Low-drain transistor audio oscillator. Proc. of the IRE. 40: 1385-1395. 1952.
- _____. Transistor amplifier -- cutoff frequency. Proc. of the IRE. 40: 1481-1483. 1952.
- Tønnesen, Tor H. On the distribution of transistor action. Proc. Phys. Soc. (London). 65B, pt. 2: 737-738. 1952.
- Trent, Robert L. Binary counter uses two transistors. Electronics. 25: 100-101. July 1952.
- Valdes, L. B. Transistor forming effects in N-type germanium. Proc. of the IRE. 40: 445-448. 1952.
- _____. Measurement of minority carrier lifetime in germanium. Proc. of the IRE. 40: 1420-1423. 1952.

- Valdes, L. B. Effect of electrode spacing on equivalent base resistance of point-contact transistors. Proc. of the IRE. 40: 1429-1434. 1952.
- van Roosbroeck, W. Theory of the flow of electrons and holes in germanium and other semiconductors. Bell Sys. Tech. Jour. 29: 560-607. 1950.
- _____. Transport of added carriers in a homogeneous semiconductor. Phys. Rev. 91: 282-289. 1953.
- Wallace, R. L., Jr. and Peitenpol, W. J. Some circuit properties and applications of N-P-N transistors. Bell Sys. Tech. Jour. 30: 530-563. 1951.
- _____ and Raisbeck, G. Duality as a guide to transistor circuit design. Bell Sys. Tech. Jour. 30: 381-418. 1951.
- _____ and Schimpf, L. G. A junction transistor tetrode for high-frequency use. Proc. of the IRE. 40: 1395-1400. 1952.
- _____, _____ and Dickten, E. High-frequency transistor tetrode. Electronics. 26: 112-113. Jan. 1953.
- Waltz, M. C. On some transients in the pulse response of point-contact germanium diodes. Proc. of the IRE. 40: 1483-1487. 1952.
- Webster, W. M., Eberhard, E. and Barton, L. E. Some novel circuits for three-terminal semiconductor amplifiers. RCA Review. 10: 5-17. 1949.
- Zadek, L. A. A note on the analysis of vacuum tube and transistor circuits. Proc. of the IRE. 41: 989-992. 1953.

VIII. ACKNOWLEDGMENTS

The author wishes to express his appreciation to Professor W. L. Cassell for his excellent suggestions and criticisms; to Professor M. S. Coover whose encouragement and assistance made this thesis possible; and to Professors L. W. Von Tersch and R. L. Doty for their interest and cooperation.

IX. APPENDICES

APPENDIX A. MEASUREMENT OF TRANSISTOR
NETWORK COEFFICIENTS

The equivalent circuit parameters for an active network can be found through the network coefficients. This may be shown by considering the general representation of an active network such as shown in Figure 1.

The mesh equations for an active circuit are

$$E_1 = r_{11}I_1 + r_{12}I_2 , \quad (140)$$

and

$$E_2 = r_{21}I_1 + r_{22}I_2 . \quad (141)$$

If the circuit is driven at the input terminals and I_2 is made zero, these equations can be written as

$$r_{11} = \frac{E_1}{I_1} , \quad (142)$$

and

$$r_{21} = \frac{E_2}{I_1} . \quad (143)$$

Equation (142) shows that r_{11} is the input impedance of the circuit. Equation (143) shows that r_{21} can be found by taking the ratio of output voltage to input current. Similarly, if the input and output terminals are interchanged, the network coefficients r_{12} and r_{22} can be found.

Although this method can be applied to transistors, it has one serious disadvantage. In order to make I_1 or I_2 zero and still maintain the d-c bias current, a high quality current source is needed. For use with junction transistors, this current source must have more than ten megohms internal impedance.

The network coefficients pertaining to the nodal-derived equivalent circuit can also be measured. This proves to be more convenient for transistors which are short-circuit stable.

The nodal equations for an active network can be written as

$$I_1 = g_{11}E_1 + g_{12}E_2, \quad (144)$$

and

$$I_2 = g_{21}E_1 + g_{22}E_2. \quad (145)$$

If the circuit is driven at the input terminals and E_2 is made zero, equations (144) and (145) may be written as

$$g_{11} = \frac{I_1}{E_1}, \quad (146)$$

and

$$g_{21} = \frac{I_2}{E_1}. \quad (147)$$

E_2 may be made zero by supplying the d-c bias voltage by means of a battery. Equation (146) shows that g_{11} is the input admittance of the circuit. Equation (147) shows that g_{21} can be found by taking the ratio of output current to input voltage. Similarly, if the input and output terminals are interchanged, g_{12} and g_{22} can be measured.

Many bridge-type circuits have been used to measure the network coefficients pertaining to the nodal equations. In particular, the General Radio Type 561-D Vacuum-Tube Bridge¹ is well suited. Since this bridge was designed primarily for vacuum tubes, all controls are labeled with vacuum-tube parameters. It is therefore necessary to compare vacuum-tube and transistor network coefficients.

The nodal equation for the grounded cathode vacuum tube under normal operation can be written as

$$I_1 = (0)E_1 + (0)E_2, \quad (148)$$

and

$$I_2 = g_m E_1 + g_p E_2. \quad (149)$$

Here E_1 is the grid to cathode voltage and E_2 is the plate to cathode voltage. The network coefficients are defined as follows:

g_m -- mutual conductance ,

and

g_p -- plate conductance .

For inverted service, the nodal equations for the vacuum tube are

$$I_1 = g_g E_1 + g_{pg} E_2, \quad (150)$$

and

$$I_2 = (0)E_1 + (0)E_2. \quad (151)$$

¹General Radio Company. Operating instructions for type 561-D vacuum-tube bridge. 275 Massachusetts Avenue, Cambridge 39, Massachusetts. Jan. 1953.

The network coefficients of equation (150) are defined as follows:

ξ_g -- grid conductance ,

and

ξ_{pg} -- inverse mutual conductance .

The network coefficients of equations (149) and (150) are not all measured directly. What is measured directly is as follows:

$r_g = \frac{1}{\xi_g}$ -- grid resistance ,

ξ_{pg} -- inverse mutual conductance ,

$\mu_{pg} = \frac{\xi_{pg}}{\xi_g}$ -- inverse amplification factor ,

$r_p = \frac{1}{\xi_p}$ -- plate resistance ,

ξ_m -- mutual conductance ,

and

$\mu = \frac{\xi_m}{\xi_p}$ -- amplification factor .

Table VI

Vacuum-Tube and Transistor Network Coefficients

Bridge Setting	Coefficient Measured
r_g	$1/\xi_{11}$
ξ_{pg}	ξ_{12}
μ_{pg}	ξ_{12}/ξ_{11}
r_p	$1/\xi_{22}$
ξ_m	ξ_{21}
μ	ξ_{21}/ξ_{22}

Table VI demonstrates the use of the General Radio Bridge for measuring transistor network coefficients. The first column shows the bridge setting or that which would be measured if a vacuum tube were being used. The second column shows what is measured when a transistor is used. These can easily be obtained by comparing equations (149) and (150) with equations (144) and (145) respectively.

APPENDIX B. RETURN DIFFERENCE AND
SENSITIVITY FOR TRANSISTORS

Special consideration must be given to transistors in defining return difference and sensitivity¹.

The nodal-derived equivalent circuit of Figure 25 can be considered to be driven by the voltage source of internal resistance R_g

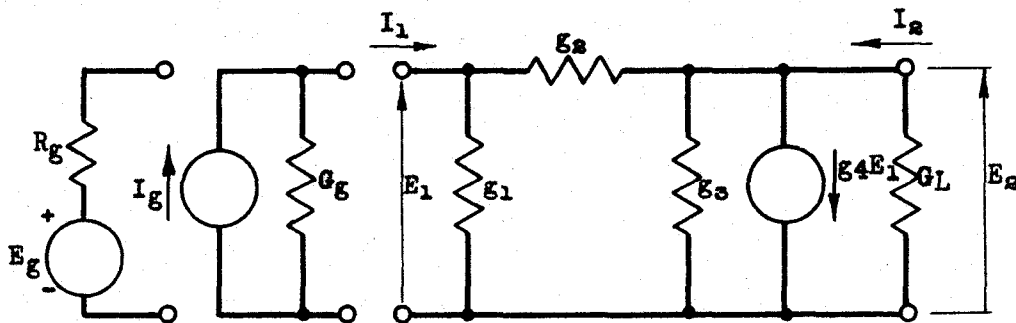


Figure 25. Nodal-derived equivalent circuit

¹H. W. Bode. Network Analysis and Feedback Amplifier-design.
1st ed. D. Van Nostrand Company, Inc. 1945.

or the current source with shunting conductance G_g . The use of either source will produce the same end result when calculating return difference if

$$R_g = \frac{1}{G_g} , \quad (152)$$

and

$$I_g = E_g G_g . \quad (153)$$

It should be noted that for feedback to exist

$$R_g \neq 0 . \quad (154)$$

This is also true for vacuum-tube circuits which employ conductance between plate and grid analogous to g_p in Figure 25. The fact that gain changes when g_p is varied does not necessarily signify feedback.

A physical explanation of return difference can be given by use of the circuit of Figure 26. Here it is assumed that the nodal

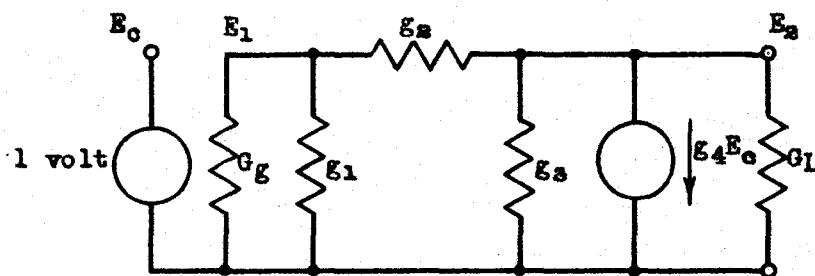


Figure 26. Nodal-derived circuit with return difference

point E_0 controls the active element similar to the grid in the vacuum tube. If unit voltage is applied to this control point, the circuit equations are

$$0 = E_1(G_g + g_1 + g_2) - E_2 g_2, \quad (155)$$

and

$$-E_4 = -E_1 g_2 + E_2(g_2 + g_3 + G_L). \quad (156)$$

Solving equations (155) and (160) for E_1 gives

$$E_1 = -\frac{E_4 g_2}{\Delta_n^0}, \quad (157)$$

where Δ_n^0 is the normal circuit determinant Δ_n with g_4 set to zero¹. It can be written as

$$\Delta_n^0 = (G_g + g_1 + g_2)(g_2 + g_3 + G_L) - g_2^2. \quad (158)$$

The voltage difference between the control point and the node at E_1 is

$$E_0 - E_1 = \frac{\Delta_n}{\Delta_n^0} = F_n, \quad (159)$$

where F_n is the return difference for the nodal-derived equivalent circuit.

¹The normal circuit determinant is obtained when the nodes at E_0 and E_1 are connected together.

The sensitivity for g_4 is defined as

$$S_n \equiv \frac{1}{\frac{dA}{dg_4} \cdot \frac{g_4}{A}} . \quad (160)$$

Here voltage gain or current gain can be used providing they are properly interpreted. Referring to Figure 25, voltage gain must be defined as

$$A_E \equiv \frac{E_a}{E_g} . \quad (161)$$

This makes it necessary to use the voltage source when computing this gain. If current gain is used, it must be defined as

$$A_I \equiv - \frac{I_a}{I_g} . \quad (162)$$

The current source must be used when computing A_I .

The above expressions for gain seem very reasonable when considering what is implied by the sensitivity expression. Stated in words, the reciprocal of sensitivity is ratio of the percentage change in gain (due to a change in g_4) to the percentage change in g_4 . Realizing that E_1 or I_1 does not remain constant when g_4 varies, gain expressions must be written to include the source characteristics. Equations (161) and (162) do this by taking into account the source resistance or conductance respectively. Evaluating equations (161) and (162) for the circuit of Figure 25, we have

$$A_E = \frac{G_G(\xi_R - \xi_4)}{\Delta_n}, \quad (163)$$

and

$$A_I = \frac{G_L(\xi_R - \xi_4)}{\Delta_n}. \quad (164)$$

Using either of the above expressions, the sensitivity defined by equation (160) becomes

$$S_n = \left[\frac{\xi_4 - \xi_R}{\xi_4} \right] \left[\frac{\Delta_n}{(G_G + \xi_1 + \xi_R)(\xi_R + \xi_3 + G_L)} \right]. \quad (165)$$

Similarly, for the mesh-derived equivalent circuit of Figure 27, the use of either source will produce the same expression for return difference when equations (152) and (153) hold.

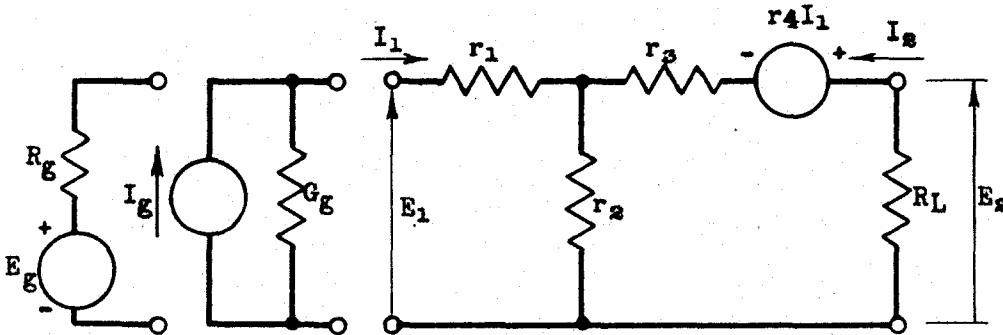


Figure 27. Mesh-derived equivalent circuit

A physical explanation of return difference for the mesh-derived circuit can be made using the circuit of Figure 28. Here the branch carrying the current I_0 is assumed to control the active element. If unit current is applied to this branch as shown, the circuit equations can be written as

$$0 = (R_g + r_1 + r_2)I_1 + r_2 I_2, \quad (166)$$

and

$$-r_4 = r_2 I_1 (r_2 + r_3 + R_L) I_2. \quad (167)$$

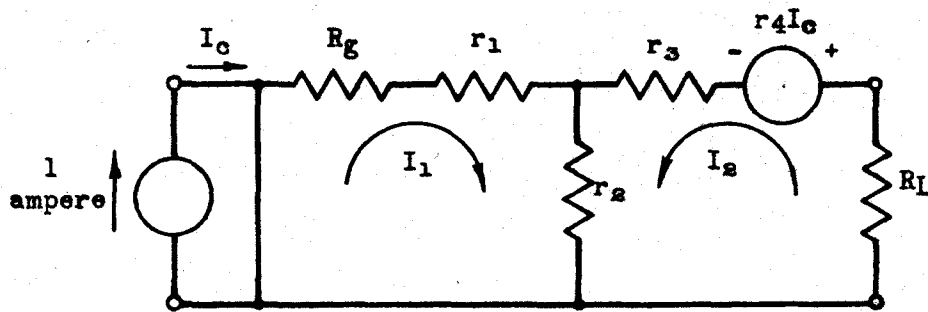


Figure 28. Mesh-derived equivalent circuit with return difference

Solving equations (166) and (167) for I_1 gives

$$I_1 = \frac{r_2 r_4}{\Delta_m}, \quad (168)$$

where Δ_m^0 is the normal circuit determinant Δ_m with r_4 set to zero¹.

It can be written as

$$\Delta_m^0 = (R_g + r_1 + r_2)(r_2 + r_3 + R_L) - r_2^2. \quad (169)$$

From this, the difference between the currents I_0 and I_1 can be written as

$$I_0 - I_1 = \frac{\Delta_m^0}{\Delta_m} = F_M, \quad (170)$$

where F_M is the return difference for the mesh-derived equivalent circuit.

In calculating sensitivity, either of the following gains must be used:

$$A_E = \frac{E_2}{E_g} = \frac{(r_2 + r_4)R_L}{\Delta_m}, \quad (171)$$

or

$$A_I = -\frac{I_2}{I_g} = \frac{(r_2 + r_4)R_g}{\Delta_m}. \quad (172)$$

The sensitivity for r_4 can then be written as

$$S = \frac{1}{\frac{dA}{dr_4} \cdot \frac{r_4}{A}} = \left[\frac{r_2 + r_4}{r_4} \right] \left[\frac{\Delta_m}{(R_g + r_1 + r_2)(r_2 + r_3 + R_L)} \right]. \quad (173)$$

¹The normal circuit determinant is obtained when the branch shorting the current source is opened.

The above definitions of return difference and sensitivity are concerned with the active element only. For the nodal-derived circuit, the return difference and sensitivity for g_4 was defined. This corresponds to the return difference and sensitivity for g_m in vacuum-tube circuits. For the mesh-derived circuit, the return difference and sensitivity for r_4 was defined. There is no analogous case for this in vacuum tubes.